JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON THURSDAY 23rd JANUARY 2025) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The value of $\int_{e^2}^{e^4}\frac{1}{x}\left(\frac{e^{\left((\log_e x)^2+1\right)^{-1}}}{e^{\left((\log_e x)^2+1\right)^{-1}}+e^{\left((6-\log_e x)^2+1\right)^{-1}}}\right)dx$ is

(1) $\log_e 2$ (2) 2

(3) 1 (4) e^2

Ans. **(3)**

Sol. Let $\ln x = t \Rightarrow \frac{dx}{x} = dt$

$$I=\int_2^4 \frac{e^{\frac{1}{1+t^2}}}{e^{\frac{1}{1+t^2}}+e^{\frac{1}{1+(6-t)^2}}}dt$$

$$I=\int_2^4 \frac{e^{\frac{1}{1+(6-t)^2}}}{e^{\frac{1}{1+(6-t)^2}}+e^{\frac{1}{1+t^2}}}dt$$

$$2I=\int_2^4 dt=(t)_2^4=4-2=2$$

$I = 1$

2. Let $I(x)=\int\frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$.

If $I(37) - I(24) = \frac{1}{4}\left(\frac{1}{b^{\frac{1}{13}}}-\frac{1}{c^{\frac{1}{13}}}\right)$, b, c ∈ $\mathbb{N}$, then 3(b + c) is equal to

(1) 40 (2) 39

(3) 22 (4) 26

Ans. **(2)**

Sol. $I(x)=\int\frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$

Put $\frac{x-11}{x+15}=t \Rightarrow \frac{26}{(x+5)^2}dx=dt$

$$I(x)=\frac{1}{26}\int\frac{dt}{t^{11/13}}=\frac{1}{26}\cdot\frac{t^{2/13}}{2/13}$$

$$I(x)=\frac{1}{4}\left(\frac{x-11}{x+15}\right)^{2/13}+C$$

$$I(37)-I(24)=\frac{1}{4}\left(\frac{26}{52}\right)^{2/13}-\frac{1}{4}\left(\frac{13}{39}\right)^{2/13}$$

$$=\frac{1}{4}\left(\frac{1}{2^{2/13}}-\frac{1}{3^{2/13}}\right)$$

$$=\frac{1}{4}\left(\frac{1}{4^{1/13}}-\frac{1}{9^{1/13}}\right)$$

∴ b = 4, c = 9

3(b + c) = 39

3. If the function

$$f(x)=\begin{cases}\frac{2}{x}\left\{\sin(k_1+1)x+\sin(k_2-1)x\right\} & , \quad x<0\\ 4 & , \quad x=0\\ \frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right) & , \quad x>0\end{cases}$$

is continuous at x = 0, then $k_1^2 + k_2^2$ is equal to

(1) 8 (2) 20

(3) 5 (4) 10

Ans. **(4)**

Sol. $\lim_{x\to0^-}\frac{2}{x}\left\{\sin(k_1+1)x+\sin(k_2-1)x\right\}=4$

$\Rightarrow 2(k_1 + 1) + 2(k_2 - 1) = 4$

$\Rightarrow k_1 + k_2 = 2$

$$\Rightarrow \lim_{x\to0^+}\frac{2}{x}\ln\left(\frac{2+k_1x}{2+k_2x}\right)=4$$

$$\Rightarrow \lim_{x\to0^+}\frac{1}{x}\ln\left(1+\frac{(k_1-k_2)x}{2+k_2x}\right)=2$$

$$\Rightarrow \frac{k_1-k_2}{2}=2$$

$\Rightarrow k_1 - k_2 = 4$

$\therefore k_1 = 3, k_2 = -1$

$k_1^2 + k_2^2 = 9 + 1 = 10$

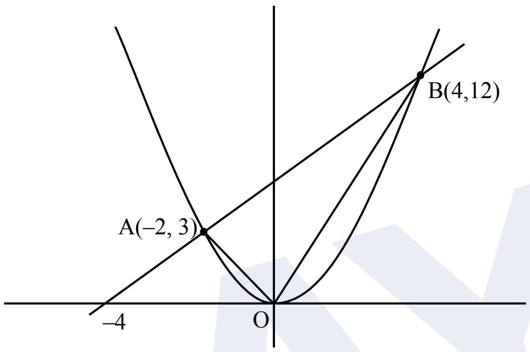
4. If the line 3x – 2y + 12 = 0 intersects the parabola $4y = 3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to

(1) $\tan^{-1}\left(\frac{11}{9}\right)$ (2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{3}{2}\right)$

(3) $\tan^{-1}\left(\frac{4}{5}\right)$ (4) $\tan^{-1}\left(\frac{9}{7}\right)$

Ans. (4)

Sol.



$3x - 2y + 12 = 0$

$4y = 3x^2$

$\therefore 2(3x + 12) = 3x^2$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2, 4$

$m_{OA} = -3/2, m_{OB} = 3$

$$\tan\theta = \left(\frac{\frac{-3}{2} - 3}{1 - \frac{9}{2}}\right) = \frac{9}{7}$$

$\theta = \tan^{-1}\left(\frac{9}{7}\right)$ (angle will be acute)

5. Let a curve y = f(x) pass through the points (0, 5) and $(\log_e 2, k)$. If the curve satisfies the differential equation $2(3 + y)e^{2x}dx - (7 + e^{2x})dy = 0$, then k is equal to

(1) 16 (2) 8

(3) 32 (4) 4

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{2(3+y).e^{2x}}{7+e^{2x}}$

$$\frac{dy}{dx} - \frac{2y.e^{2x}}{7+e^{2x}} = \frac{6.e^{2x}}{7+e^{2x}}$$

$$\text{I.F.} = e^{-\int \frac{2e^{2x}}{7+e^{2x}}dx} = \frac{1}{7+e^{2x}}$$

$$\therefore \; y.\frac{1}{7+e^{2x}} = \int \frac{6e^{2x}}{\left(7+3^{2x}\right)^2}dx$$

$$\frac{y}{7+e^{2x}} = \frac{-3}{7+e^{2x}} + C$$

$(0, 5) \Rightarrow \frac{5}{8} = \frac{-3}{8} + C \Rightarrow C = 1$

$\therefore y = -3 + 7 + e^{2x}$

$y = e^{2x} + 4$

$\therefore k = 8$

6. Let $f(x) = \log_e x$ and $g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$.

Then the domain of fog is

(1) $\mathbb{R}$ (2) $(0, \infty)$

(3) $[0, \infty)$ (4) $[1, \infty)$

Ans. (1)

Sol. f(x) = lnx

$$g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$$

$D_g \in R$

$D_f \in (0, \infty)$

For $D_{fog} \Rightarrow g(x) > 0$

$$\frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1} > 0$$

$\Rightarrow x^4 - 2x^3 + 3x^2 - 2x + 2 > 0$

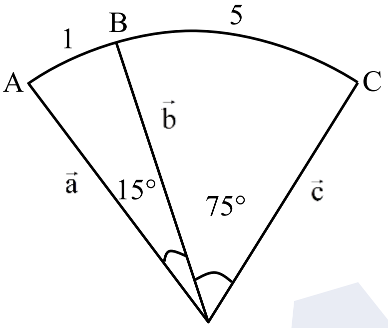
Clearly x < 0 satisfies which are included in option (1) only.

7. Let the arc AC of a circle subtend a right angle at the centre O. If the point B on the arc AC, divides the arc AC such that $\frac{\text{length of arc AB}}{\text{length of arc BC}} = \frac{1}{5}$, and $\overrightarrow{OC} = \alpha\overrightarrow{OA} + \beta\overrightarrow{OB}$, then $\alpha = \sqrt{2}\left(\sqrt{3}-1\right)\beta$ is equal to

(1) $2-\sqrt{3}$
(2) $2\sqrt{3}$
(3) $5\sqrt{3}$
(4) $2+\sqrt{3}$

Ans. (1)

Sol.



$\vec{c} = \alpha\vec{a} + \beta\vec{b}$ ....(1)

$\vec{a}.\vec{c} = \alpha\vec{a}.\vec{a} + \beta\vec{b}.\vec{a}$

$0 = \alpha + \beta \cos 15°$ ....(2)

(1) ⇒ $\vec{b}.\vec{c} = \alpha\vec{a}.\vec{b} + \beta\vec{b}.\vec{b}$

⇒ $\cos 75° = \alpha \cos 15° + \beta$ ....(3)

(2) & (3) ⇒ $\cos 75° = -\beta \cos^2 15° + \beta$

$$\beta = \frac{\cos 75°}{\sin^2 15°} = \frac{1}{\sin 15°} = \frac{2\sqrt{2}}{\sqrt{3}-1}$$

$$(2) \Rightarrow \alpha = \frac{-\cos 15°}{\sin 15°} = \frac{-\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)}$$

$$\therefore \vec{c} = \frac{-\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)}\vec{a} + \left(\frac{2\sqrt{2}}{\sqrt{3}-1}\right)\vec{b}$$

Now

$$\alpha + \sqrt{2}\left(\sqrt{3}-1\right)\beta = \frac{-\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)} + \frac{\sqrt{2}\left(\sqrt{3}-1\right).2\sqrt{2}}{\sqrt{3}-1}$$

$$= \frac{-\left(\sqrt{3}+1\right)^2}{2} + 4$$

$$= \frac{-3-1-2\sqrt{3}+8}{2}$$

$$= 2-\sqrt{3}$$

8. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

(1) –1200
(2) –1080
(3) –1020
(4) –120

Ans. (2)

Sol. a = 3

$$S_4 = \frac{1}{5}(S_8 - S_4)$$

$\Rightarrow 5S_4 = S_8 - S_4$

$\Rightarrow 6S_4 = S_8$

$$\Rightarrow 6.\frac{4}{2}\left[2\times3+\left(4-1\right)\times d\right]$$

$$= \frac{8}{2}\left[2\times3+\left(8-1\right)d\right]$$

$\Rightarrow 12(6 + 3d) = 4(6 + 7d)$

$\Rightarrow 18 + 9d = 6 + 7d$

$\Rightarrow d = -6$

$$S_{20} = \frac{20}{2}\left[2\times3+\left(20-1\right)\left(-6\right)\right]$$

$= 10\,[6 - 114]$

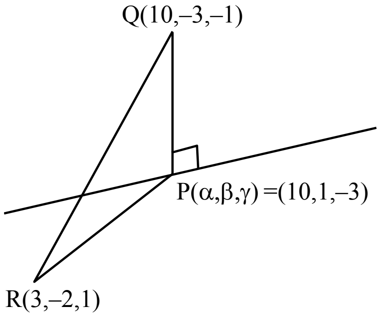
$= -1080$

9. Let P be the foot of the perpendicular from the point Q(10, –3, –1) on the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2}$. Then the area of the right angled triangle PQR, where R is the point (3, –2, 1), is

(1) $9\sqrt{15}$
(2) $\sqrt{30}$
(3) $8\sqrt{15}$
(4) $3\sqrt{30}$

Ans. (4)

Sol.



$$\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}=\lambda$$

⇒ 7λ + 3, – λ + 2, – 2 λ – 1

dr's of QP ⇒

7 λ – 7, – λ + 5, – 2 λ

Now

(7 λ – 7) . 7 – (–λ + 5) + (2 λ).2 = 0

54 λ – 54 = 0 ⇒ λ = 1

∴ P = (10, 1, – 3)

$$\overrightarrow{PQ}=-4\hat{j}+2\hat{k}$$

$$\overrightarrow{PR}=-7\hat{i}-3\hat{j}+4\hat{k}$$

$$\text{Area}=\left|\frac{1}{2}\begin{Vmatrix} i & j & k \\ 0 & -4 & 2 \\ -7 & -3 & 4 \end{Vmatrix}\right| = 3\sqrt{30}$$

10. Let $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}$, z ∈ ℂ, be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points (0, 0), C and (α, 0) is 11 square units, then α^2 equals

(1) 100 (2) 50

(3) $\frac{121}{25}$ (4) $\frac{81}{25}$

Ans. **(1)**

Sol. $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}$

$$\left|\frac{\bar{z}-i}{\bar{z}+\frac{i}{2}}\right|=\frac{2}{3}$$

$3|x - iy - i| = 2\,|x - iy + \frac{i}{2}|$

$9(x^2 + (y+1)^2) = 4\,(x^2 + (y - 1/3)^2)$

$9x^2 + 9y^2 + 18\,y + 9 = 4x^2 + 4y^2 - 4y + 1$

$5x^2 + 5y^2 + 22y + 8 = 0$

$$x^2+y^2+\frac{22}{5}y+\frac{8}{5}=0$$

centre ⇒ $(0, -\frac{11}{5})$

$$\left|\frac{1}{2}\begin{Vmatrix} 0 & 0 & 1 \\ 0 & -11/5 & 1 \\ \alpha & 0 & 1 \end{Vmatrix}\right|=11$$

$$\Rightarrow\left(-\frac{11}{5}\alpha\right)^2=(11\times 2)^2$$

⇒ $\alpha^2 = 100$

11. Let R = {(1, 2), (2, 3), (3,3)} be a relation defined on the set {1, 2, 3, 4}. Then the minimum number of elements, needed to be added in R so the R becomes an equivalence relation, is :

(1) 10 (2) 8

(3) 9 (4) 7

Ans. **(4)**

Sol. A = {1, 2, 3, 4}

For relation to be reflexive

R = {(1,2), (2, 3), (3,3)}

Minimum elements added will be

(1,1), (2,2), (4,4) (2,1) (3,2) (3,2) (3,1) (1,3)

∴ Minimum number of elements = 7

Option : (4)

12. The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is

(1) 34000 (2) 37000

(3) 36000 (4) 35000

Ans. **(3)**

Sol. DAUGHTER

Total words = 8!

Total words in which vowels are together = 6! × 3!

words in which all vowels are not together

= 8! – 6! × 3!

= 6! [56 – 6]

= 720 × 50

= 36000

Ans.(3)

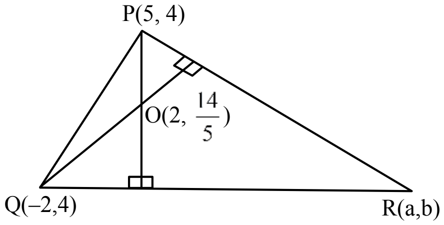
13. Let the area of a ΔPQR with vertices P(5, 4), Q(–2, 4) and R(a, b) be 35 square units. If its orthocenter and centroid are $O\left(2, \frac{14}{5}\right)$ and C(c, d) respectively, then c + 2d is equal to

(1) $\frac{7}{3}$ (2) 3

(3) 2 (4) $\frac{8}{3}$

Ans. **(2)**

Sol.



Equation of lines QR = 5x + 2y + 2 = 0

Equation of lines PR = 10x –3y – 38 = 0

$\therefore$ Point R (2, –6)

$$\text{Centroid} = \left(\frac{5-2+2}{3}, \frac{4+4-6}{3}\right)$$

$$= \left(\frac{5}{3}, \frac{2}{3}\right)$$

$$c + 2d = \frac{5}{3} + \frac{4}{3} = 3$$

14. If $\frac{\pi}{2} \le x \le \frac{3\pi}{4}$, then $\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{13}\sin x\right)$ is equal to

(1) $x - \tan^{-1}\frac{4}{3}$ (2) $x - \tan^{-1}\frac{5}{12}$

(3) $x + \tan^{-1}\frac{4}{5}$ (4) $x + \tan^{-1}\frac{5}{12}$

Ans. **(2)**

Sol. $\frac{\pi}{2} \le x \le \frac{3\pi}{4}$

$$\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{12}\sin x\right)$$

$\cos^{-1}(\cos x \cos\alpha + \sin x \sin\alpha)$

$\cos^{-1}(\cos(x-\alpha))$

$\Rightarrow x - \alpha$ because $x - \alpha \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\Rightarrow x - \tan^{-1}\frac{5}{12}$.

15. The value of $(\sin 70°)(\cot 10° \cot 70° - 1)$ is

(1) 1 (2) 0

(3) 3/2 (4) 2/3

Ans. **(1)**

Sol. $\sin 70^{o}$ $(\cot 10^{o} \cot 70^{o} - 1)$

$$\Rightarrow \frac{\cos(80°)}{\sin 10} = 1$$

16. Marks obtains by all the students of class 12 are presented in a frequency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12. If the number of students whose marks are less than 12 is 18, then the total number of students is

(1) 48 (2) 44

(3) 40 (4) 52

Ans. **(2)**

Sol. $\text{medain} = \ell + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$

$$= 12 + \left(\frac{\frac{N}{2} - 18}{12}\right) \times 6 = 14$$

$$\Rightarrow \left(\frac{\frac{N}{2} - 18}{12}\right) \times 6 = 2$$

$$\frac{N}{2} - 18 = 4 \quad \Rightarrow \quad N = 44$$

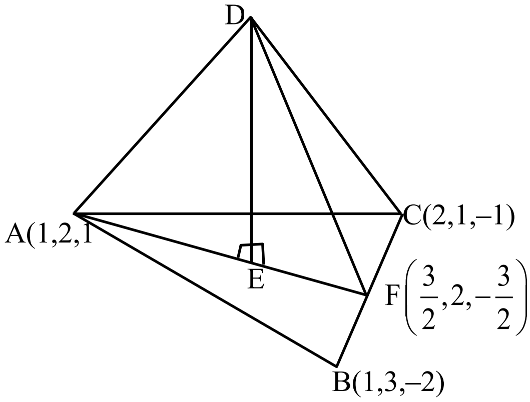
17. Let the position vectors of the vertices A, B and C of a tetrahedron ABCD be $\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} - \hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is

(1) $\frac{1}{2}\left(\hat{i} + 4\hat{j} + 7\hat{k}\right)$ (2) $\frac{1}{12}\left(7\hat{i} + 4\hat{j} + 3\hat{k}\right)$

(3) $\frac{1}{6}\left(12\hat{i} + 12\hat{j} + \hat{k}\right)$ (4) $\frac{1}{6}\left(7\hat{i} + 12\hat{j} + \hat{k}\right)$

Ans. **(4)**

Sol.



Area of ΔABC = $\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$=\frac{1}{2}\left|5\hat{i}+3\hat{j}+\hat{k}\right|=\frac{1}{2}\sqrt{35}$

volume of tetrahedron

$=\frac{1}{3}\times \text{Base area}\times h=\frac{\sqrt{805}}{6\sqrt{2}}$

$\frac{1}{3}\times\frac{1}{2}\sqrt{35}\times h=\frac{\sqrt{805}}{6\sqrt{2}}$

$h=\sqrt{\frac{23}{2}}$

$AE^2=AD^2-DE^2=\frac{13}{18}\quad\therefore\quad AE=\sqrt{\frac{13}{18}}$

$\overrightarrow{AE}=|AE|.\left(\frac{\hat{i}-5\hat{k}}{\sqrt{26}}\right)$

$=\sqrt{\frac{13}{18}}.\left(\frac{\hat{i}-5\hat{k}}{\sqrt{26}}\right)$

$=\sqrt{\frac{13}{18}}.\left(\frac{\hat{i}-5\hat{k}}{\sqrt{26}}\right)=\frac{\hat{i}-5\hat{k}}{6}$

P.V. of E $=\frac{\hat{i}-5\hat{k}}{6}+\hat{i}+2\hat{j}+\hat{k}=\frac{1}{6}\left(7\hat{i}+12\hat{j}+\hat{k}\right)$

18. If A, B and (adj(A^{-1}) + adj(B^{-1})) are non-singular matrices of same order, then the inverse of A(adj(A^{-1}) + adj(B^{-1}))$^{-1}$B, is equal to

(1) $AB^{-1}+A^{-1}B$ (2) adj(B^{-1}) + adj(A^{-1})

(3) $\frac{1}{|AB|}$(adj(B) +adj(A)) (4) $\frac{AB^{-1}}{|A|}+\frac{BA^{-1}}{|B|}$

Ans. **(3)**

Sol. $\left[A\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)^{-1}.B\right]^{-1}$

$B^{-1}.\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right).A^{-1}$

$B^{-1}\text{adj}\left(A^{-1}\right)A^{-1}+B^{-1}\left(\text{adj}\left(B^{-1}\right)\right).A^{-1}$

$B^{-1}\left|A^{-1}\right|I+\left|B^{-1}\right|IA^{-1}$

$\frac{B^{-1}}{|A|}+\frac{A^{-1}}{|B|}$

$\Rightarrow\frac{\text{adj}B}{|B||A|}+\frac{\text{adj}A}{|A||B|}$

$=\frac{1}{|A||B|}\left(\text{adj}B+\text{adj}A\right)$

19. If the system of equations
$(\lambda-1)x+(\lambda-4)y+\lambda z=5$
$\lambda x+(\lambda-1)y+(\lambda-4)z=7$
$(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$
has infinitely many solutions, then $\lambda^2+\lambda$ is equal to

(1) 10 (2) 12

(3) 6 (4) 20

Ans. **(2)**

Sol. $(\lambda-1)x+(\lambda-4)y+\lambda z=5$

$\lambda x+(\lambda-1)y+(\lambda-4)z=7$

$(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$

For infinitely many solutions

$$D=\begin{vmatrix}\lambda-1 & \lambda-4 & \lambda\\ \lambda & \lambda-1 & \lambda-4\\ \lambda+1 & \lambda+2 & -(\lambda+2)\end{vmatrix}=0$$

$(\lambda-3)(2\lambda+1)=0$

$$D_x=\begin{vmatrix}5 & \lambda-4 & \lambda\\ 7 & \lambda-1 & \lambda-4\\ 9 & \lambda+2 & -(\lambda+2)\end{vmatrix}=0$$

$2(3-\lambda)(23-2\lambda)=0$

$\lambda=3$

$\therefore\ \lambda^2+\lambda=9+3=12$

20. One die has two faces marked 1, two faces marked 2, one face marked 3 and one face marked 4. Another die has one face marked 1, two faces marked 2, two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5, when both the dice are thrown together, is

(1) $\frac{1}{2}$ (2) $\frac{3}{5}$

(3) $\frac{2}{3}$ (4) $\frac{4}{9}$

Ans. **(1)**

Sol. a = number or dice 1

b = number on dice 2

(a,b) = (1,3), (3,1), (2,2), (2,3), (3,2), (1,4), (4,1)

Required probability

$$=\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{2}{6}$$

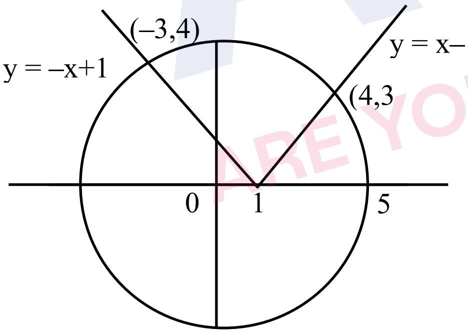
$$=\frac{18}{36}=\frac{1}{2}$$

SECTION-B

21. If the area of the larger portion bounded between the curves $x^2 + y^2 = 25$ and $y = |x - 1|$ is $\frac{1}{4}(b\pi + c)$, $b, c \in \mathbb{N}$, then b + c is equal to ________

Ans. **(77)**

Sol.



$x^2 + y^2 = 5$

$x^2 + (x - 1)^2 = 25 \Rightarrow x = 4$

$x^2 + (-x + 1)^2 = 5 \Rightarrow x = -3$

$$A = 25\pi - \int_{-3}^{4}\sqrt{25-x^2}\,dx + \frac{1}{2}\times4\times4+\frac{1}{2}\times3\times3$$

$$A = 25\pi + \frac{25}{2} - \left[\frac{x}{2}\sqrt{25-x^2}+\frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-3}^{4}$$

$$A = 25\pi + \frac{25}{2} - \left[6+\frac{25}{2}\sin^{-1}\frac{4}{5}+6+\frac{25}{2}\sin^{-1}\frac{3}{5}\right]$$

$$A = 25\pi + \frac{1}{2} - \frac{25}{2}.\frac{\pi}{2} \;;\; A = \frac{75\pi}{4}+\frac{1}{2}$$

$$A = \frac{1}{4}(75\pi + 2)$$

b = 75, c= 2

b + c = 75 + 2 = 77

22. The sum of all rational terms in the expansion of $(1 + 2^{1/3} + 3^{1/2})^6$ is equal to ________

Ans. **(612)**

Sol. $\left(1+2^{\frac{1}{3}}+3^{\frac{1}{2}}\right)^6$

$$=\frac{\underline{|6}}{\underline{|r_1}\,\underline{|r_2}\,\underline{|r_3}}(1)^{r_1}(2)^{\frac{r_2}{3}}(3)\frac{r_3}{2}$$

r_1	r_2	r_3
6	0	0
4	0	2
2	0	4
0	0	6
3	3	0
1	3	2
0	6	0

$$\text{sum} = \frac{\underline{|6}}{\underline{|6}\,\underline{|0}\,\underline{|0}}+\frac{\underline{|6}}{\underline{|4}\,\underline{|0}\,\underline{|2}}(3)+\frac{\underline{|6}}{\underline{|2}\,\underline{|0}\,\underline{|4}}(3)^2+\frac{\underline{|6}}{\underline{|0}\,\underline{|0}\,\underline{|6}}(3)^3$$

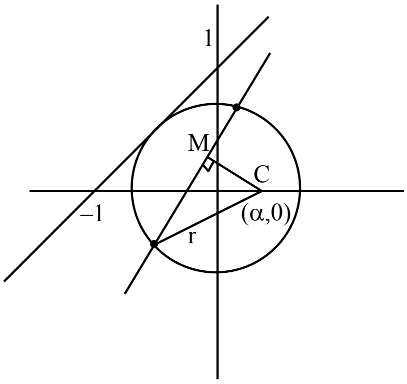
$$+\frac{\underline{|6}}{\underline{|3}\,\underline{|3}\,\underline{|0}}(2)+\frac{\underline{|6}}{\underline{|1}\,\underline{|3}\,\underline{|2}}(2)^1(3)^1+\frac{\underline{|6}}{\underline{|0}\,\underline{|6}\,\underline{|0}}(2)^2$$

= 1 + 45 + 135 + 27 + 40 + 360 + 4 = 612

23. Let the circle C touch the line x – y + 1 = 0, have the centre on the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line –3x + 2y = 1. Let H be the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then $2\alpha^2 + 3\beta^2$ is equal to ________

Ans. **(19)**

Sol.



$x - y + 1 = 0$; $p = r$

$$\left|\frac{\alpha - 0 + 1}{\sqrt{2}}\right| = r \Rightarrow (\alpha + 1)^2 = 2r^2 \text{ .... (1)}$$

now $\left(\frac{-3\alpha + 0 - 1}{\sqrt{9+4}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2 = r^2$

$\Rightarrow (3\alpha + 1)^2 + 4 = 13\, r^2$ ......(2)

(1) & (2) $\Rightarrow (3\alpha + 1)^2 + 4 = 13\,\frac{(\alpha+1)^2}{2}$

$\Rightarrow 18\alpha^2 + 12\alpha + 2 + 8 = 13\alpha^2 + 26\alpha + 13$

$\Rightarrow 5\alpha^2 - 14\alpha - 3 = 0$

$\Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0$; $\Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0$

$\Rightarrow \alpha = \frac{-1}{5}, 3$

$\therefore \quad r = 2\sqrt{2}$

How $\alpha e = 3$ and $2\alpha = 4\sqrt{2}$

$\alpha^2 e^2 = 9 \Rightarrow \alpha = 2\sqrt{2} \Rightarrow \alpha^2 = 8$

$\alpha^2\left(1 + \frac{\beta^2}{\alpha^2}\right) = 9$

$\alpha^2 + \beta^2 = 9$

$\therefore \beta^2 = 1$

$\therefore 2\alpha^2 + 3\beta^2 = 2\,(8) + 3(1) = 19$

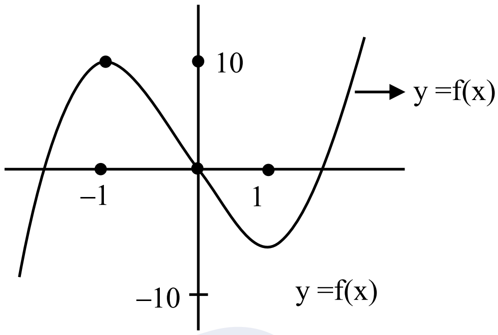
24. If the set of all values of a, for which the equation $5x^3 - 15x - a = 0$ has three distinct real roots, is the interval (α, β), then $\beta - 2\alpha$ is equal to ________

Ans. (30)

Sol. $5x^3 - 15x - a = 0$

$f(x) = 5x^3 - 15x$

$f'(x) = 15x^2 - 15 = 15(x-1)(x+1)$



$a \in (-10, 10)$

$\alpha = -10, \beta = 10$

$\beta - 2\alpha = 10 + 20 = 30$

25. If the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, where $a + c = 15$ and $b = \frac{36}{5}$, then $a^2 + c^2$ is equal to ________

Ans. (117)

Sol. $a(b-c)\,x^2 + b\,(c-a)\,x + c(a-b) = 0$

$x = 1$ is root $\therefore$ other root is 1

$\alpha + \beta = -\frac{b(c-a)}{a(b-c)} = 2$

$\Rightarrow -bc + ab = 2ab - 2ac$

$\Rightarrow 2ac = ab + bc$

$\Rightarrow 2ac = b(a + c)$

$\Rightarrow 2ac = 15b$ ... (1)

$\Rightarrow 2ac = 15\left(\frac{36}{5}\right) = 108$

$\Rightarrow ac = 54$

$a + c = 15$

$a^2 + c^2 + 2ac = 225$

$a^2 + c^2 = 225 - 108 = 117$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON THURSDAY 23rd JANUARY 2025) **TIME : 9:00 AM TO 12:00 NOON**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

26. Regarding self-inductance :

A : The self-inductance of the coil depends on its geometry.

B : Self-inductance does not depend on the permeability of the medium.

C : Self-induced e.m.f. opposes any change in the current in a circuit.

D : Self-inductance is electromagnetic analogue of mass in mechanics.

E : Work needs to be done against self-induced e.m.f. in establishing the current.

Choose the correct answer from the options given below:

(1) A, B, C, D only (2) A, C, D, E only

(3) A, B, C, E only (4) B, C, D, E only

Ans. **(2)**

Sol. Self inductance of coil

$$L = \frac{\mu_0 N^2 A}{2\pi R}$$

27. A light hollow cube of side length 10 cm and mass 10g, is floating in water. It is pushed down and released to execute simple harmonic oscillations. The time period of oscillations is $y\pi \times 10^{-2}$ s, where the value of y is

(Acceleration due to gravity, $g = 10$ m/s^2, density of water $= 10^3$ kg/m^3)

(1) 2 (2) 6

(3) 4 (4) 1

Ans. **(1)**

Sol. $a^2 x\ \rho g = m a_{net}$

$$\frac{L^2 \rho g}{m} x = a_{net}$$

$$T = 2\pi \sqrt{\frac{m}{L^2 \rho g}}$$

where m = 10g, L = 10 cm, $\rho = 1000$ kg/m^3

28. Given below are two statements:

Statement-I : The hot water flows faster than cold water.

Statement-II : Soap water has higher surface tension as compared to fresh water.

In the light above statements, choose the ***correct*** answer from the options given below

(1) Statement-I is false but Statement II is true

(2) Statement-I is true but Statement II is false

(3) Both Statement-I and Statement-II are true

(4) Both Statement-I and Statement-II are false

Ans. **(2)**

Sol. Hot water is less viscous then cold water.

Surfactant reduces surface tension.

29. A sub-atomic particle of mass 10^{-30} kg is moving with a velocity 2.21×10^6 m/s. Under the matter wave consideration, the particle will behave closely like ______. ($h = 6.63 \times 10^{-34}$ J.s)

(1) Infra-red radiation (2) X-rays

(3) Gamma rays (4) Visible radiation

Ans. **(2)**

Sol. $$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{10^{-30} \times 2.21 \times 10^6}$$

$= 3 \times 10^{-10}$m

Hence particle will behave as x-ray.

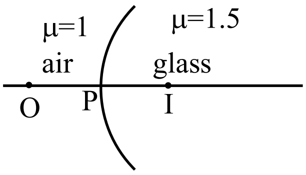
30. A spherical surface of radius of curvature R, separates air from glass (refractive index = 1.5). The centre of curvature is in the glass medium. A point object 'O' placed in air on the optic axis of the surface, so that its real image is formed at 'I' inside glass. The line OI intersects the spherical surface at P and PO = PI. The distance PO equals to-

(1) 5R (2) 3R

(3) 2R (4) 1.5R

Ans. **(1)**

Sol.



PO = u = –x

PI = v = x

PO = PI

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{1.5}{x}+\frac{1}{x}=\frac{1}{2R}$$

$$\frac{5}{2x}=\frac{1}{2R}$$

X = 5R

31. A radioactive nucleus n_2 has 3 times the decay constant as compared to the decay constant of another radioactive nucleus n_1. If initial number of both nuclei are the same, what is the ratio of number of nuclei of n_2 to the number of nuclei of n_1, after one half-life of n_1?

(1) 1/4 (2) 1/8

(3) 4 (4) 8

Ans. **(1)**

Sol. $N_2 = N_0e^{-3\lambda t}$

$N_1 = N_0e^{-\lambda t}$

$$\frac{N_2}{N_1}=e^{-2\lambda t}$$

$$t_{\text{half life of} N_1} t=\frac{\ln 2}{\lambda}\ {}_{n}1$$

$$\frac{N_0}{2}=N_0e^{-\lambda t}$$

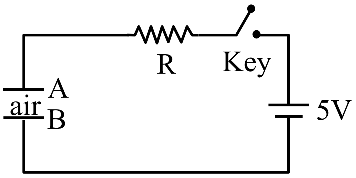
$\lambda t = \ln 2$

$$t=\frac{\ln 2}{\lambda}$$

$$=e^{-2\lambda\frac{\ln 2}{\lambda}}$$

$$\frac{N_2}{N_1}=\frac{1}{4}$$

32. Identify the valid statements relevant to the given circuit at the instant when the key is closed.



A. There will be no current through resistor R.

B. There will be maximum current in the connecting wires.

C. Potential difference between the capacitor plates A and B is minimum.

D. Charge on the capacitor plates is minimum.

Choose the correct answer from the options given below :

(1) C, D only (2) B, C, D only

(3) A, C only (4) A, B, D only

Ans. **(2)**

Sol. Initially capacitor behave as a short circuit.

So current will be maximum.

Charge on capacitor will be zero.

Potential difference across capacitor will be zero.

33. The position of a particle moving on x-axis is given by $x(t) = A \sin t + B \cos^2 t + Ct^2 + D$, where t is time. The dimension of $\frac{ABC}{D}$ is-

(1) L (2) $L^3 T^{-2}$

(3) $L^2 T^{-2}$ (4) L^2

Ans. **(3)**

Sol. Dimension [x(t)] = [L]

[A] = [L]

[B] = [L]

[C] = [LT^{-2}]

[D] = [L]

$$\left[\frac{ABC}{D}\right]=\left[\frac{L\times L\times LT^{-2}}{L}\right]=[L^2T^{-2}]$$

34. Match the **List-I** with **List-II**

List-I		**List-II**	
A.	Pressure varies inversely with volume of an ideal gas.	I.	Adiabatic process
B.	Heat absorbed goes partly to increase internal energy and partly to do work.	II.	Isochoric process
C.	Heat is neither absorbed nor released by a system	III.	Isothermal process
D.	No work is done on or by a gas	IV.	Isobaric process

Choose the ***correct*** answer from the options given below :

(1) A–I, B–IV, C–II, D–III

(2) A–III, B–I, C–IV, D–II

(3) A–I, B–III, C–II, D–IV

(4) A–III, B–IV, C–I, D–II

Ans. (4)

Sol. $A \rightarrow P \propto \frac{1}{V}$

$\Rightarrow PV = \text{constant}$

$\Rightarrow\ nRT = \text{const.} \Rightarrow T = \text{const.}$

Hence Isothermal III

$B \rightarrow IV$

$W \neq 0, \Delta U \neq 0, \Delta Q \neq 0$ [only isobaric]

$C \rightarrow I\ \Delta Q = 0$ Adiabatic

$D \rightarrow II\ w = 0$ Isochoric

III IV I II

35. Consider a moving coil galvanometer (MCG) :

A : The torsional constant in moving coil galvanometer has dimensions $[ML^2T^{-2}]$

B : Increasing the current sensitivity may not necessarily increase the voltage sensitivity.

C : If we increase number of turns (N) to its double (2N), then the voltage sensitivity doubles.

D : MCG can be converted into an ammeter by introducing a shunt resistance of large value in parallel with galvanometer.

E : Current sensitivity of MCG depends inversely on number of turns of coil.

Choose the correct answer from the options given below :

(1) A, B only
(2) A, D, only
(3) B, D, E only
(4) A, B, E only

Ans. (1)

Sol. (A) $\tau = C\theta \Rightarrow [ML^2T^{-2}] = [C][1]$

(B) C.S $= \frac{\theta}{I} = \frac{BNA}{C}$;

$\text{V.S.} = \frac{BNA}{RC}$ [R also depends on ‘N’]

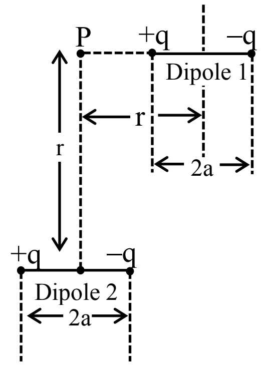
(C) $\text{V.S.} \propto \frac{NAB}{CR}$ $\qquad R \rightarrow NR$

(D) False [Theory]

(E) E [False] $\qquad C.S \propto N$

$\Rightarrow \because C.S. = \frac{NAB}{C}$

36. A point particle of charge Q is located at P along the axis of an electric dipole 1 at a distance r as shown in the figure. The point P is also on the equatorial plane of a second electric dipole 2 at a distance r. The dipoles are made of opposite charge q separated by a distance 2a. For the charge particle at P not to experience any net force, which of the following correctly describes the situation?

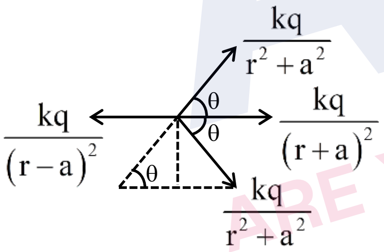


(1) $\frac{a}{r}-20$ (2) $\frac{a}{r}\sim 10$

(3) $\frac{a}{r}\sim 0.5$ (4) $\frac{a}{r}\sim 3$

Ans. **(4)**

Sol.



$$\frac{kq}{(r-a)^2}=\frac{kq}{(r+a)^2}+\frac{2kq}{(r^2+a^2)}\cos\theta$$

$$\frac{1}{(r-a)^2}=\frac{1}{(r+a)^2}+\frac{2a}{(r^2+a^2)^{\frac{3}{2}}}$$

$$\frac{1}{(r-a)^2}-\frac{1}{(r+a)^2}=\frac{2a}{(r^2+a^2)^{\frac{3}{2}}}$$

$$\frac{4ra}{(r^2-a^2)^2}=\frac{2a}{(r^2+a^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{2r}{(r^2-a^2)^2}=\frac{1}{(r^2+a^2)^{\frac{3}{2}}}$$

$$\frac{4r^2}{(r^2-a^2)^4}=\frac{1}{(r^2+a^2)^3}$$

$$\Rightarrow 4r^2(r^2+a^2)^3=(r^2-a^2)^4$$

$$4r^8\left(1+\frac{a^2}{r^2}\right)^3=r^8\left(1-\frac{a^2}{r^2}\right)^4$$

$$4\left(1+\frac{a^2}{r^2}\right)^3=\left(1-\frac{a^2}{r^2}\right)^4$$

Exact value cannot be solved in exam for this equation to be true

$$\left|\frac{a}{r}\right|>1 \Rightarrow a>r$$

But point charge Q lies between charges of dipole 1 hence electric field cannot be zero.

There for it should be bonus.

But by solving from mathematical software we are getting a/r ≈ 3.

37. A gun fires a lead bullet of temperature 300K into a wooden block. The bullet having melting temperature of 600 K penetrates into the block and melts down. If the total heat required for the process is 625 J, then the mass of the bullet is ___ grams.

(Latent heat of fusion of lead = 2.5×10^4 JKg^{-1} and specific heat capacity of lead = 125 $\text{JKg}^{-1}\,\text{K}^{-1}$)

(1) 20 (2) 15

(3) 10 (4) 5

Ans. **(3)**

Sol. $625 = ms\Delta T + mL$

$625 = m[125\times300+2.5\times10^4]$

$625 = m[37500+25000]$

$625 = m[62500]$

$$m=\frac{1}{100}\text{kg}$$

M = 10 grams

38. What is the lateral shift of a ray refracted through a parallel-sided glass slab of thickness 'h' in terms of the angle of incidence 'i' and angle of refraction 'r', if the glass slab is placed in air medium ?

(1) $\frac{h\tan(i-r)}{\tan r}$ (2) $\frac{h\cos(i-r)}{\sin r}$

(3) h (4) $\frac{h\sin(i-r)}{\cos r}$

Ans. (4)

Sol. Formula base

$$\frac{h\sin(i-r)}{\cos r}$$

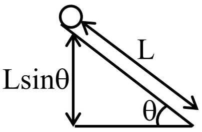
39. A solid sphere of mass 'm' and radius 'r' is allowed to roll without slipping from the highest point of an inclined plane of length 'L' and makes an angle 30° with the horizontal. The speed of the particle at the bottom of the plane is v_1. If the angle of inclination is increased to 45° while keeping L constant. Then the new speed of the sphere at the bottom of the plane is v_2. The ratio of $v_1^2 : v_2^2$ is

(1) $1 : \sqrt{2}$ (2) 1 : 3

(3) 1 : 2 (4) $1 : \sqrt{3}$

Ans. (1)

Sol.



using WET

$W_g = k_f - k_i$

$Mg\ L\ \sin\theta = k_f - k_i$

K.E. in pure rolling $\frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

$$= \frac{1}{2}mV^2 + \frac{1}{2}\times\frac{2}{5}mR^2\frac{V^2}{R^2}$$

$$\frac{7}{10}mV^2$$

$$mgL\sin\theta = \frac{7}{10}mV_f^2 - 0$$

$$V_f^2 \propto \sin\theta$$

$$\left(\frac{V_1}{V_2}\right)^2 = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin 30°}{\sin 45°} = \frac{1}{\sqrt{2}}$$

40. Refer to the circuit diagram given in the figure, which of the following observation are correct?

A. Total resistance of circuit is 6 Ω.

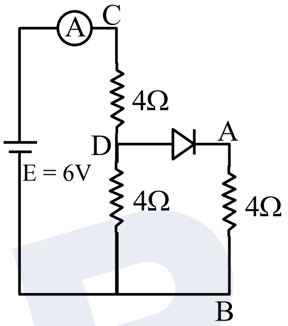
B. Current in Ammeter is 1A

C. Potential across AB is 4 Volts.

D. Potential across CD is 4 Volts.

E. Total resistance of the circuit is 8Ω.

Choose the correct answer from the options given below:

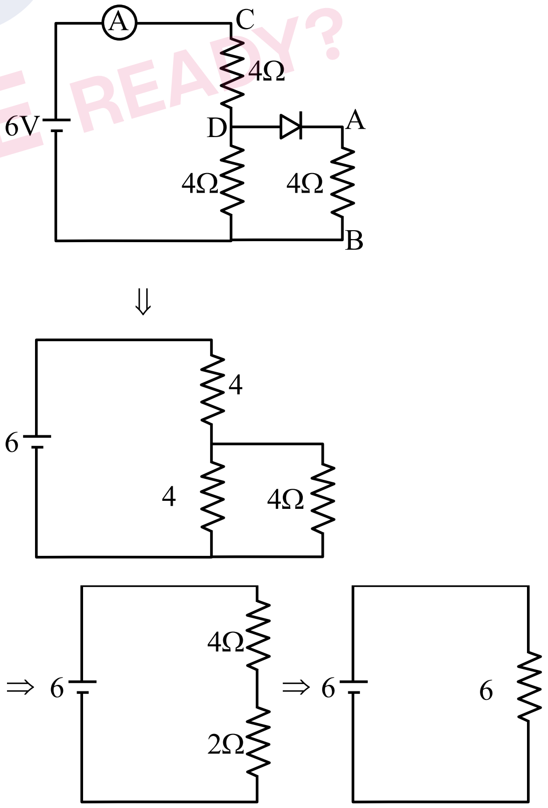


(1) A, B and D only (2) A, C and D only

(3) B, C and E only (4) A, B and C only

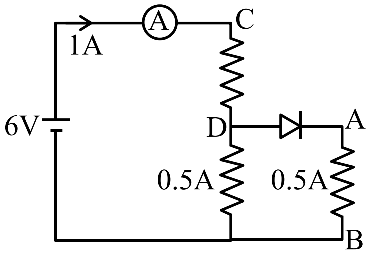
Ans. (1)

Sol.



Current through ammeter = 1 A

$R_{net} = 6\Omega$



$V_{AB} = 0.5 \times 4 = 2$ volt

$V_{CD} = 1 \times 4 = 4$ volt

A, B & D are correct

41. The electric flux is $\phi = \alpha\sigma + \beta\lambda$

where λ and σ are linear and surface charge density, respectively, $\left(\frac{\alpha}{\beta}\right)$ represents

(1) charge

(2) electric field

(3) displacement

(4) area

Ans. (3)

Sol. $\phi = \alpha\sigma + \beta\lambda$

$[\phi] = [\alpha\sigma] = [\beta\lambda]$

$$[\alpha] = \frac{[\phi]}{[\sigma]} \qquad \left[\frac{\alpha}{\beta}\right] = \frac{[\lambda]}{[\sigma]}$$

$$[\beta] = \frac{[\phi]}{[\lambda]} = \frac{[Q/L]}{[Q/\text{Area}]} = \left[\frac{\text{Area}}{\text{Length}}\right]$$

$$\left[\frac{\alpha}{\beta}\right] = L$$

42. Given a thin convex lens (refractive index μ_2), kept in a liquid (refractive index μ_1, $\mu_1 < \mu_2$) having radii of curvature $|R_1|$ and $|R_2|$. Its second surface is silver polished. Where should an object be placed on the optic axis so that a real and inverted image is formed at the same place ?

(1) $\dfrac{\mu_1|R_1|.|R_2|}{\mu_2(|R_1|+|R_2|)-\mu_1|R_1|}$

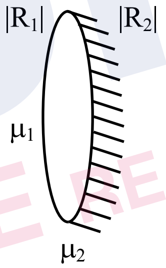
(2) $\dfrac{\mu_1|R_1|.|R_2|}{\mu_2(|R_1|+|R_2|)-\mu_1|R_2|}$

(3) $\dfrac{\mu_1|R_1|.|R_2|}{\mu_2(2|R_1|+|R_2|)-\mu_1\sqrt{|R_1|.|R_2|}}$

(4) $\dfrac{(\mu_2+\mu_1)|R_1|}{(\mu_2-\mu_1)}$

Ans. (2)

Sol.



$$\frac{1}{f_{eq}} = \frac{2}{f_L} - \frac{1}{f_m}$$

$$f_m = -\frac{|R_2|}{2}$$

$$\frac{1}{f_L} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{f_{eq}} = 2\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left(\frac{R_1+R_2}{R_1R_2}\right) + \frac{2}{R_2}$$

$$= \frac{2}{R_2}\left[\frac{(\mu_2-\mu_1)(R_1+R_2)+\mu_1R_1}{\mu_1R_1}\right]$$

$$= \frac{2}{R_2}\left[\frac{\mu_2R_1+\mu_2R_2-\mu_1R_1-\mu_1R_2+\mu_1R_1}{\mu_1R_1}\right]$$

$$\frac{1}{f_{eq}}=\frac{2[\mu_2R_1+\mu_2R_2-\mu_1R_2]}{\mu_1R_1R_2}$$

For same size of image

$u = 2f$

$$u=\frac{\mu_1R_1R_2}{\mu_2R_1+\mu_2R_2-\mu_1R_2}$$

43. The electric field of an electromagnetic wave in free space is

$\vec{E} = 57\cos[7.5\times10^{6}t-5\times10^{-3}(3x+4y)]$ $\left(4\hat{i}-3\hat{j}\right)$ N/C.

The associated magnetic field in Tesla is-

(1) $\vec{B}=\frac{57}{3\times10^{8}}\cos\ [7.5\times10^{6}t-5\times10^{-3}(3x+4y)]\left(5\hat{k}\right)$

(2) $\vec{B}=\frac{57}{3\times10^{8}}\cos\ [7.5\times10^{6}t-5\times10^{-3}(3x+4y)]\left(\hat{k}\right)$

(3) $\vec{B}=-\frac{57}{3\times10^{8}}\cos\ [7.5\times10^{6}t-5\times10^{-3}(3x+4y)]\left(5\hat{k}\right)$

(4) $\vec{B}=-\frac{57}{3\times10^{8}}\cos\ [7.5\times10^{6}t-5\times10^{-3}(3x+4y)]\left(\hat{k}\right)$

Ans. **(3)**

Sol. $\vec{K}=3\hat{i}+4\hat{j}$

$$\hat{K}=\frac{3\hat{i}+4\hat{j}}{5}$$

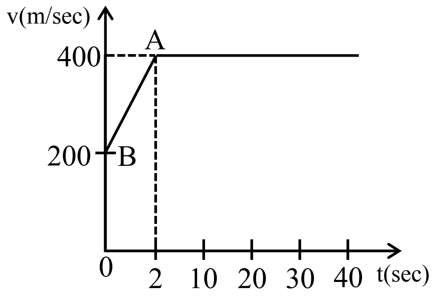
$$\hat{E}=\frac{4\hat{i}-3\hat{j}}{5}$$

$\hat{B}=\hat{K}\times\hat{E}$

$\hat{B}=-\hat{Z}$

$$B_0=\frac{E_0}{C}=\frac{57}{3\times10^{8}}$$

44. The motion of an airplane is represented by velocity-time graph as shown below. The distance covered by airplane in the first 30.5 second is ____ km.



(1) 9	(2) 6
(3) 3	(4) 12

Ans. **(4)**

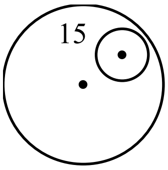
Sol. Total Area under curve.

45. Consider a circular disc of radius 20 cm with centre located at the origin. A circular hole of a radius 5 cm is cut from this disc in such a way that the edge of the hole touches the edge of the disc. The distance of centre of mass of residual or remaining disc from the origin will be-

(1) 2.0 cm	(2) 0.5 cm
(3) 1.5 cm	(4) 1.0 cm

Ans. **(4)**

Sol.



mass of disc = m

mass of cut part $=\frac{m}{16}$

$$X_{com}=\frac{m\times0-\frac{m}{16}\times15}{m-\frac{m}{16}}$$

= 1 cm.

SECTION-B

46. A positive ion A and a negative ion B has charges 6.67×10^{-19} C and 9.6×10^{-10} C, and masses 19.2×10^{-27} kg and 9×10^{-27} kg respectively. At an instant, the ions are separated by a certain distance r. At that instant the ratio of the magnitudes of electrostatic force to gravitational force is $P \times 10^{-13}$, where the value of P is ____.

(Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ $\mathrm{Nm^2C^{-1}}$ and universal gravitational constant as 6.67×10^{-11} $\mathrm{Nm^2\,kg^{-2}}$)

Ans. **(BONUS)**

Sol. $$\frac{9 \times 10^9 \times 6.67 \times 10^{-19} \times 9.6 \times 10^{-10}}{6.67 \times 10^{-11} \times 19.2 \times 10^{-27} \times 9 \times 10^{-27}}$$

$$\frac{1}{2} \times 10^{45}$$

Charge is not integral multiple of electron.

47. Two particles are located at equal distance from origin. The position vectors of those are represented by $\vec{A} = 2\hat{i} + 3n\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4p\hat{k}$, respectively. If both the vectors are at right angle to each other, the value of n^{-1} is____.

Ans. (3)

Sol. $\vec{A}.\vec{B} = 0$

$4 - 6\,n + 8p = 0$

$|\vec{A}| = |\vec{B}|$

$4 + 9n^2 + 4 = 4 + 4 + 16p^2$

$9n^2 = 16p^2$

$P = + \frac{3}{4} n$

$4 - 6n \pm 6n = 0$

$12n = 4$

$n = \frac{1}{3}$

48. An ideal gas initially at 0°C temperature, is compressed suddenly to one fourth of its volume. If the ratio of specific heat at constant pressure to that at constant volume is 3/2, the change in temperature due to the thermodynamics process is____K.

Ans. **(273)**

Sol. $\gamma = \frac{3}{2}$

$Tv^{\gamma - 1} = C$

$273\ V_0^{0.5} = T\left(\frac{V_0}{4}\right)^{0.5}$

$T = 273 \times 2 = 546$

$\Delta T = 273$

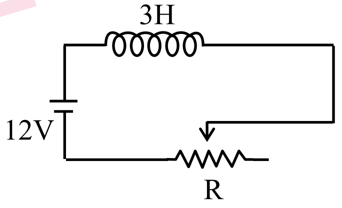
49. A force $f = x^2 y\hat{i} + y^2\hat{j}$ acts on a particle in a plane x + y = 10. The work done by this force during a displacement from (0, 0) to (4m, 2m) is _____ Joule (round off to the nearest integer)

Ans. **(152)**

Sol. $$\int_0^4 x^2(10 - x)dx + \int_0^2 y^2 dy$$

$$= \left[\frac{10x^3}{3} - \frac{x^4}{4}\right]_0^4 + \left[\frac{y^3}{3}\right]_0^2 = \frac{640}{3} - 64 + \frac{8}{3} = 152$$

50.



In the given circuit the sliding contact is pulled outwards such that electric current in the circuit changes at the rate of 8 A/s. At an instant when R is 12 Ω, the value of the current in the circuit will be ____A.

Ans. **(3)**

Sol. $$\varepsilon - \frac{LdI}{dt} - IR = 0$$

$12 - 3 \times (-8) - I \times 12 = 0$

$I = 3$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON THURSDAY 23rd JANUARY 2025) **TIME : 9 : 00 AM TO 12 : 00 NOON**

CHEMISTRY

TEST PAPER WITH SOLUTIONS

SECTION-A

51. The element that does not belong to the same period of the remaining elements (modern periodic table) is:

(1) Palladium
(2) Iridium
(3) Osmium
(4) Platinum

Sol. **(1)**

Palladium ⇒ 5th period

Iridium, Osmium, Platinum ⇒ 6th Period

52. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H atom is suitable for this ?

Given: Rydberg constant

$R_H = 10^5\ cm^{-1}$, $h = 6.6 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m/s)

(1) Paschen series, $\infty \to 3$
(2) Lyman series, $\infty \to 1$
(3) Balmer series, $\infty \to 2$
(4) Paschen series, $5 \to 3$

Sol. **(1)**

$\lambda = 900$ nm H-atom (Z = 1)

$= 9 \times 10^{-5}$ cm

$R_H = 10^5\ cm^{-1}$

$$\text{Ryderg eq.} = \frac{1}{\lambda} = R_H Z^2 \times \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{\lambda \times R_H} = \frac{1}{n_1^2} - \frac{1}{n_2^2}$$

$$\Rightarrow \frac{1}{9 \times 10^{-5} cm \times 10^5 cm^{-1}} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{1}{9}$$

It is possible when $n_1 = 3$, $n_2 = \infty$

Possible series : $\infty \to 3$

53. The **incorrect** statements among the following is

(1) PH_3 shows lower proton affinity than NH_3.
(2) PF_3 exists but NF_5 does not.
(3) NO_2 can dimerise easily.
(4) SO_2 can act as an oxidizing agent, but not as a reducing agent.

Sol. **(4)**

SO_2 can oxidise as well as reduce.

Hence it can act as both oxidising and reducing agent.

54. $CrCl_3.xNH_3$ can exist as a complex. 0.1 molal aqueous solution of this complex shows a depression in freezing point of 0.558ºC. Assuming 100% ionisation of this complex and coordination number of Cr is 6, the complex will be

(Given $K_f = 1.86$ K kg mol^{-1})

(1) $[Cr(NH_3)_6] Cl_3$
(2) $[Cr(NH_3)_4Cl_2] Cl$
(3) $[Cr(NH_3)_5Cl] Cl_2$
(4) $[Cr(NH_3)_3Cl_3]$

Sol. **(3)**

Given : $\Delta T_f = 0.558°C$

$$k_f = 1.86 \frac{K \times kg}{mol}$$

0.1 m aq. sol.

$\Rightarrow \Delta T_f = i \times k_f \times m$

$\Rightarrow 0.558 = i \times 1.86 \times 0.1$

$\Rightarrow i = 3$

55. $FeO_4^{2-} \xrightarrow{+2.0\,V} Fe^{3+} \xrightarrow{0.8\,V} Fe^{2+} \xrightarrow{-0.5\,V} Fe^{0}$

In the above diagram, the standard electrode potentials are given in volts (over the arrow).

The value of $E^{\ominus}_{FeO_4^{2-}/Fe^{2+}}$ is

(1) 1.7 V (2) 1.2 V

(3) 2.1 V (4) 1.4 V

Sol. (1)

$$\begin{tikzcd}
FeO_4^{-2} \arrow[r, "E_1^o=2V", "n_1=3"'] \arrow[rr, bend right, "E_4^o = ?"'] & Fe^{+3} \arrow[r, "E_2^o=0.8V", "n_2=1"'] & Fe^{+2} \arrow[r, "E_3^o=-.05V"] & Fe
\end{tikzcd}$$

$$n_4 = 4$$

$$\Delta G_4^o = \Delta G_1^o + \Delta G_2^o$$

$$\Rightarrow -n_4FE_4^o = -n_1FE_1^0 - n_2FE_2^o$$

$$\Rightarrow +4E_4^o = 3\times2+(1\times0.8)$$

$$\Rightarrow E_4^o = \frac{6.8}{4}V$$

$$\Rightarrow \boxed{E_4^o = 1.7V}$$

56. Match the **LIST-I** with **LIST-II**

LIST-I **Name reaction**		**LIST-II** **Product obtainable**	
A.	Swarts reaction	I.	Ethyl benzene
B.	Sandmeyer's reaction	II.	Ethyl iodide
C.	Wurtz Fittig reaction	III.	Cyanobenzene
D.	Finkelstein reaction	IV.	Ethyl fluoride

Choose the ***correct*** answer from the options given below:

(1) A-II, B-III, C-I, D-IV
(2) A-IV, B-I, C-III, D-II
(3) A-IV, B-III, C-I, D-II
(4) A-II, B-I, C-III, D-IV

Sol. (3)

LIST-I **Name reaction**		**LIST-II** **Product obtainable**	
A.	Swarts reaction	I.	$Et\text{-}I \xrightarrow[DMF]{KF} Et\text{-}F$
B.	Sandmeyer's reaction	II.	$PhN_2^{\oplus}Cl^- \xrightarrow{CuCN/KCN} PhCN + N_2$
C.	Wurtz Fittig reaction	III.	$Ph{-}Cl + EtCl \xrightarrow[ether]{Na}$ $Ph{-}Et + Ph{-}Ph + Et{-}Et$
D.	Finkelstein reaction	IV.	$Et{-}Cl \xrightarrow[acetone]{NaI} Et\text{-}I + NaCl$

57. Given below are two statements:

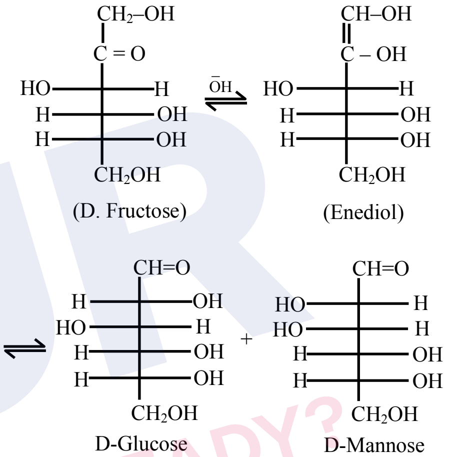
Statement I: Fructose does not contain an aldehydic group but still reduces Tollen's reagent

Statement II : In the presence of base, fructose undergoes rearrangement to give glucose.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false
(4) Statement I is true but Statement II is false

Sol. (2)



58. 2.8×10^{-3} mol of CO_2 is left after removing 10^{21} molecules from its 'x' mg sample. The mass of CO_2 taken initially is

Given : $N_A = 6.02 \times 10^{23}$ mol^{-1}

(1) 196.2 mg (2) 98.3 mg

(3) 150.4 mg (4) 48.2 mg

Sol. (1)

$$(moles)_{initial} = \frac{x\times10^{-3}}{44}$$

$$(moles)_{removal} = \frac{10^{21}}{6.02\times10^{23}}$$

$$(moles)_{left} = (moles)_{initial} - (moles)_{removed}$$

$$2.8\times10^{-3} = \frac{x\times10^{-3}}{44} - \frac{10^{21}}{6.02\times10^{23}}$$

$$\Rightarrow x = 196.2 \text{ mg}$$

59. Ice at –5°C is heated to become vapor with temperature of 110°C at atmospheric pressure. The entropy change associated with this process can be obtained from :

(1) $\int_{268\text{K}}^{383\text{K}} C_p\,dT + \frac{\Delta H_{\text{melting}}}{273} + \frac{\Delta H_{\text{boiling}}}{373}$

(2) $\int_{268\text{K}}^{273\text{K}} \frac{C_{p,m}}{T}dT + \frac{\Delta H_m, \text{fusion}}{T_f} + \frac{\Delta H_{m,\text{vaporisation}}}{T_b}$

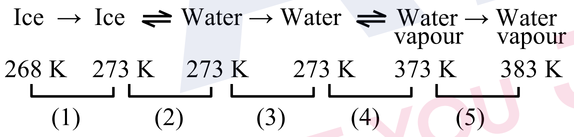
$+ \int_{273\text{K}}^{373\text{K}} \frac{C_{p,m}\,dT}{T} + \int_{373\text{K}}^{383\text{K}} \frac{C_{p,m}dT}{T}$

(3) $\int_{268\text{K}}^{383\text{K}} C_p\,dT + \frac{q_{\text{rev}}}{T}$

(4) $\int_{268\text{K}}^{273\text{K}} C_{p,m}dT + \frac{\Delta H_m, \text{fusion}}{T_f} + \frac{\Delta H_{m,\text{vaporisation}}}{T_b}$

$+ \int_{273\text{K}}^{373\text{K}} C_{p,m}dT + \int_{373\text{K}}^{383\text{K}} C_{p,m}dT$

Sol. **(2)**



$\Delta S_{\text{overall}} = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 + \Delta S_5$

$\Delta S_2 = \frac{\Delta H_{m\ \text{fusion}}}{273}$ $\quad T_f = 273\ \text{'K'}$

$\Delta S_3 = \int_{273}^{373} \frac{C_{p,m}dT}{T}$

$\Delta S_4 = \frac{\Delta H_{m\ \text{vaporisation}}}{373}$ $\quad T_b = 373\ \text{'K'}$

$\Delta S_5 = \int_{373}^{383} \frac{C_{p,m}dT}{T}$

Answer = (2)

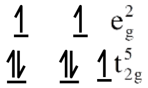
60. The d-electronic configuration of an octahedral Co(II) complex having magnetic moment of 3.95 BM is :

(1) $t_{2g}^6\,e_g^1$ (2) $t_{2g}^3\,e_g^0$

(3) $t_{2g}^5\,e_g^2$ (4) $e^4\,t_2^3$

Sol. **(3)**

$Co^{+2} = (Ar)_{18}\,3d^7\,4s°$



61. The complex that shows Facial – Meridional isomerism is

(1) $[Co(NH_3)_3Cl_3]$ (2) $[Co(NH_3)_4Cl_2]^+$

(3) $[Co(en)_3]^{3+}$ (4) $[Co(en)_2Cl_2]^+$

Sol. **(1)**

Ma_3b_3 type complexes show Facial - Meridional isomerism

(i) $[Co(NH_3)_3Cl_3] \Rightarrow Ma_3b_3$

(ii) $[Co(NH_3)_4Cl_2]^+ \Rightarrow Ma_4b_2$

(iii) $[Co(en)_3]^{3+} \Rightarrow M(AA)_3$

(iv) $[Co(en)_2Cl_2]^+ \Rightarrow M(AA)_2b_2$

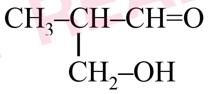
a, b, = NH_3, Cl^-

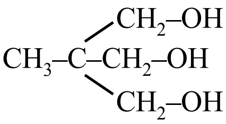
AA = en

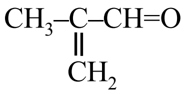
62. The major product of the following reaction is :

$CH_3CH_2CH{=}O \xrightarrow[\text{reflux}]{\text{excess HCHO, alkali}} ?$

(1) $CH_3{-}CH_2{-}CH_2{-}OH$

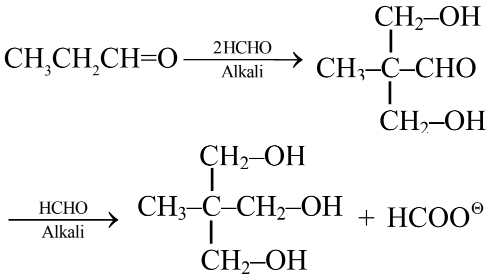
(2)

$CH_3{-}\underset{\displaystyle CH_2{-}OH}{\underset{|}{CH}}{-}CH{=}O$

(3)

$CH_3{-}C(CH_2{-}OH)_3$

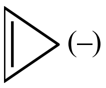
(4)

$CH_3{-}\underset{\displaystyle CH_2}{\underset{||}{C}}{-}CH{=}O$

Sol. **(3)**

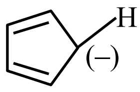
This is an example of Tollen's reaction i.e. multiple cross aldol followed by cross Cannizaro reaction



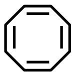
63. The correct stability order of the following species/molecules is :



p



q



r

(1) q > r > p
(2) r > q > p
(3) q > p > r
(4) p > q > r

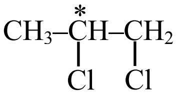
Sol. **(1)**

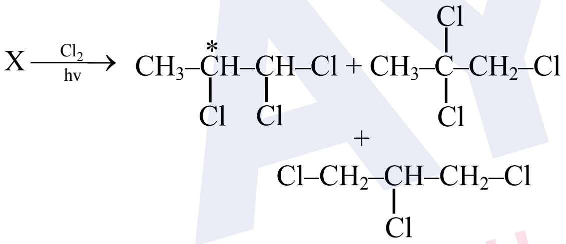
q is aromatic r is nonaromatic p is antiaromatic

64. Propane molecule on chlorination under photochemical condition gives two di-chloro products, "x" and "y". Amongst "x" and "y", "x" is an optically active molecule. How many tri-chloro products (consider only structural isomers) will be obtained from "x" when it is further treated with chlorine under the photochemical condition?

(1) 4
(2) 2
(3) 5
(4) 3

Sol. **(4)**

"X" is

$CH_3-\underset{\underset{Cl}{|}}{\overset{*}{C}H}-\underset{\underset{Cl}{|}}{CH_2}$


$X \xrightarrow[hv]{Cl_2} CH_3-\underset{\underset{Cl}{|}}{\overset{*}{C}H}-\underset{\underset{Cl}{|}}{CH}-Cl + CH_3-\overset{\overset{Cl}{|}}{\underset{\underset{Cl}{|}}{C}}-CH_2-Cl$

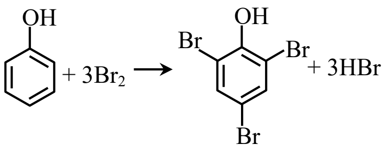
$+$

$Cl-CH_2-\underset{\underset{Cl}{|}}{CH}-CH_2-Cl$

65. What amount of bromine will be required to convert 2 g of phenol into 2, 4, 6-tribromophenol ? (Given molar mass in g mol^{-1} of C, H, O, Br are 12, 1, 16, 80 respectively)

(1) 10.22 g
(2) 6.0 g
(3) 4.0 g
(4) 20.44 g

Sol. **(1)**



Moles of phenol = $\frac{2}{94}$ = 0.021

$\therefore$ Moles of bromine = 0.021 × 3 = 0.064

$\therefore$ Mass of bromine = 0.064 × 160 = 10.22 g

66. The correct set of ions (aqueous solution) with same colour from the following is :

(1) V^{2+}, Cr^{3+}, Mn^{3+}
(2) Zn^{2+}, V^{3+}, Fe^{3+}
(3) Ti^{4+}, V^{4+}, Mn^{2+}
(4) Sc^{3+}, Ti^{3+}, Cr^{2+}

Sol. **(1)**

(1) V^{2+}(Violet), Cr^{3+}(Violet), Mn^{3+}(Violet)
(2) Zn^{2+}(Colourless), V^{3+}(Green), Fe^{3+}(Yellow)
(3) Ti^{4+}(Colourless), V^{4+}(Blue), Mn^{2+} (Pink)
(4) Sc^{3+}(Colourless), Ti^{3+}(Purple), Cr^{2+} (Blue)

67. Given below are two statements :

Statement I : In Lassaigne's test, the covalent organic molecules are transformed into ionic compounds.

Statement II : The sodium fusion extract of an organic compound having N and S gives prussian blue colour with $FeSO_4$ and $Na_4[Fe(CN)_6]$

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both **Statement I** and **Statement II** are true
(2) Both **Statement I** and **Statement II** are false
(3) **Statement I** is false but **Statement II** is true
(4) **Statement I** is true but **Statement II** is false

Sol. **(4)**

The sodium fusion extract of organic compound having N & S gives blood red colour with $FeSO_4$ and $Na_4[Fe(CN)_6]$

68. Which of the following happens when NH_4OH is added gradually to the solution containing 1M A^{2+} and 1M B^{3+} ions ?

Given : $K_{sp}[A(OH)_2] = 9 \times 10^{-10}$ and
$K_{sp}[B(OH)_3] = 27 \times 10^{-18}$ at 298 K.

(1) $B(OH)_3$ will precipitate before $A(OH)_2$
(2) $A(OH)_2$ and $B(OH)_3$ will precipitate together
(3) $A(OH)_2$ will precipitate before $B(OH)_3$
(4) Both $A(OH)_2$ and $B(OH)_3$ do not show precipitation with NH_4OH

Sol. **(1)**

Condition for precipitation $Q_{ip} > K_{sp}$

For $[A(OH)_2]$

$[A^{2+}]\,[OH^-]^2 > 9 \times 10^{-10}$

$[A^{+2}] = 1$ M

$\Rightarrow [OH^-] > 3 \times 10^{-5}$ M

For $[B(OH)_3]$

$[B^{3+}][OH^-]^3 > 27 \times 10^{-18}$

$[B^{3+}] = 1M$

$\Rightarrow [OH^-] > 3 \times 10^{-6}$ M

So, $B(OH)_3$ will precipitate before $A(OH)_2$

69. Match the **LIST-I** with **LIST-II**

LIST-I **(Classification of molecules based on octet rule)**		**LIST-II** **(Example)**	
A.	Molecules obeying octet rule	I.	NO, NO_2
B.	Molecules with incomplete octet	II.	BCl_3, $AlCl_3$
C.	Molecules with incomplete octet with odd electron	III.	H_2SO_4, PCl_5
D.	Molecules with expanded octet	IV.	CCl_4, CO_2

Choose the **correct** answer from the options given below :

(1) A-IV, B-II, C-I, D-III

(2) A-III, B-II, C-I, D-IV

(3) A-IV, B-I, C-III, D-II

(4) A-II, B-IV, C-III, D-I

Sol. **(1)**

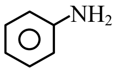
(A) A $\rightarrow$ IV

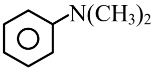
(B) B $\rightarrow$ II

(C) C $\rightarrow$ I

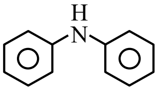
(D) D $\rightarrow$ III

70. Which among the following react with Hinsberg's reagent?

(A) 

(2) 

(C) $CH_3–NH_2$ (4) $N(CH_3)_3$

(E) 

Choose the correct answer from the options given below :

(1) B and D only (2) C and D only

(3) A, B and E only (4) A, C and E only

Sol. **(4)**

B and D are 3° amine which does not have replaceable H on N, So does not react.

SECTION-B

71. If 1 mM solution of ethylamine produces pH = 9, then the ionization constant (K_b) of ethylamine is 10^{-x}. The value of x is ________ (nearest integer).

[The degree of ionization of ethylamine can be neglected with respect to unity.]

Sol. **(7)**

Sol. $C_2H_5NH_2(aq) + H_2O \rightleftharpoons C_2H_5NH_3^+ + \overset{\ominus}{O}H$

$$\begin{array}{lcc} C = 10^{-3}\text{ M} & - & - \\ C(1 - \alpha) & C\alpha & C\alpha \\ \Rightarrow C = 10^{-3} & =10^{-5} & = 10^{-5} \end{array}$$

$$\boxed{1-\alpha \simeq 1}$$

$$\text{Given, } P^H = 9 \Rightarrow P^{OH} = 5 \Rightarrow \left[\overset{\ominus}{O}H\right] = 10^{-5}M$$

$$\text{Now, } K_b = \frac{[C_2H_5NH_3^+][\overset{\ominus}{O}H]}{[C_2H_5NH_2]}$$

$$\Rightarrow K_b = \frac{10^{-5} \times 10^{-5}}{10^{-3}} = 10^{-7}$$

72. During "S" estimation, 160 mg of an organic compound gives 466 mg of barium sulphate. The percentage of Sulphur in the given compound is ______ %.

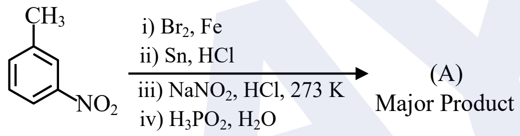
(Given molar mass in g mol^{-1} of Ba : 137, S : 32, O : 16)

Sol. **(40)**

$$\text{Millimoles of } BaSO_4 = \frac{466}{233} = 2\text{m mol}$$

$$\%S = \frac{\frac{466}{233} \times 32}{160} \times 100 = 40\%$$

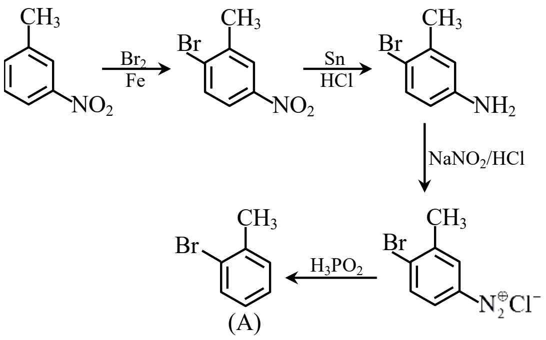
73. Consider the following sequence of reactions to produce major product (A)



Molar mass of product (A) is ______ g mol^{-1}.

(Given molar mass in g mol^{-1} of C : 12, H : 1, O : 16, Br : 80, N : 14, P : 31)

Sol. **(171)**



Molar mass of product (C_7H_7Br) (A) is 171 g mol^{-1}

74. For the thermal decomposition of $N_2O_5(g)$ at constant volume, the following table can be formed, for the reaction mentioned below :

$$2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$$

S.No.	Time/s	Total pressure / (atm)
1.	0	0.6
2.	100	'x'

x = ________ × 10^{-3} atm [nearest integer]

Given : Rate constant for the reaction is $4.606 \times 10^{-2}\ s^{-1}$.

Sol. **(900)**

NTA. **(897)**

$$K_{N_2O_5} = 2 \times 4.606 \times 10^{-2}\ S^{-1}$$

$$\begin{array}{lccc} & 2N_2O_5(g) \longrightarrow & 2N_2O_4(g) + & O_2(g) \\ P_i & 0.6 & 0 & 0 \\ P_f & 0.6 - P & P & \frac{P}{2} \end{array}$$

$$2 \times 4.606 \times 10^{-2} = \frac{2.303}{100} \log \frac{0.6}{0.6-P}$$

$$4 \log_{10} \frac{0.6}{0.6-P}$$

$$10^4 = \frac{0.6}{0.6-P}$$

$$\Rightarrow 0.6 \times 10^4 - 10^4\ P = 0.6$$

$$\Rightarrow 10^4\ P = 0.6(10^4 - 1)$$

$$P = (6000 - 0.6) \times 10^{-4}$$

$$= 5999. \times 10^{-4}$$

$$= 0.59994$$

$$P_{Total} = 0.6 + \frac{P}{2}$$

$$= 0.6 + 0.29997$$

$$= 0.89997$$

$$= 899.97 \times 10^{-3}$$

Ans. 900

Given by NTA

Given : $2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$

$t = 0$ 0.6 0 0

$t = 100s$ 0.6 - x x x/2

$$P_{Total} = 0.6 + \frac{x}{2}$$

As given in equation

$K_r = 4.606 \times 10^{-2}\ sec^{-1}$

(Here language conflict in question)

$(K_r = \frac{KA}{2}$ not considered)

$$K_r t = \ln\frac{0.6}{0.6 - x}$$

$$4.606 \times 10^{-2} \times 100 = 2.303 \log \frac{0.6}{0.6 - x}$$

$$P_{Total} = 0.6 + \frac{0.594}{2} = 0.897\ \text{atm}$$

$$= 897 \times 10^{-3}\ \text{atm}$$

75. The standard enthalpy and standard entropy of decomposition of N_2O_4 to NO_2 are 55.0 kJ mol^{-1} and 175.0 J/K/mol respectively. The standard free energy change for this reaction at 25°C in J mol^{-1} is _______ (Nearest integer)

Sol. **(2850)**

$\Delta H^o_{rxn} = 55$ kJ/mol, $T = 298$ K

$\Delta S^o_{rxn} = 175$ J/mol

$$\Delta G^o_{rxn} = \Delta H^o_{rxn} - T\Delta S^o_{rxn}$$

$$\Rightarrow \Delta G^o_{rxn} = 55000\ \text{J/mol} - 298 \times 175\ \text{J/mol}$$

$$\Rightarrow \Delta G^o_{rxn} = 55000 - 52150$$

$$\Rightarrow \Delta G^o_{rxn} = 2850\ \text{J/mol}$$