JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON FRIDAY 04th APRIL 2025) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $a > 0$. If the function $f(x) = 6x^3 - 45ax^2 + 108a^2x + 1$ attains its local maximum and minimum values at the points x_1 and x_2 respectively such that $x_1x_2 = 54$, then $a + x_1 + x_2$ is equal to :-

(1) 15

(2) 18

(3) 24

(4) 13

Ans. **(2)**

Sol. $f'(x) = 18x^2 - 90\,ax + 108\,a^2 = 0$

$x = 2a$ & $x = 3a$

$x_1 = 2a \quad x_2 = 3a$

$x_1x_2 = 54$

$6a^2 = 54$

$a = 3$

$a + x_1 + x_2$

$3 + 2 \times 3 + 3 \times 3 = 18$

option (2)

2. Let f be a differentiable function on **R** such that f(2) 1, $f'(2) = 4$. Let $\lim\limits_{x\to 0}(f(2+x))^{3/x} = e^{\alpha}$. Then the number of times the curve $y = 4x^3 - 4x^2 - 4(\alpha - 7)\,x - \alpha$ meets x-axis is :-

(1) 2 (2) 1

(3) 0 (4) 3

Ans. **(1)**

Sol. $\lim\limits_{x\to 0}(f(2+x))^{\frac{3}{x}}$

$e^{\lim\limits_{x\to 0}\frac{(f(2+x)-1)3}{x}}$

$e^{3f'(2)} = (e)^{12} = (e)^{a} \Rightarrow a = 12$

$y = 4x^3 - 4x^2 - 4\,(a - 7)\,x - a$

$y = 4x^3 - 4x^2 - 20x - 12$

roots $x = -1, -1, 3$

option (1)

3. The sum of the infinite series $\cot^{-1}\left(\frac{7}{4}\right) + \cot^{-1}\left(\frac{19}{4}\right) + \cot^{-1}\left(\frac{39}{4}\right) + \cot^{-1}\left(\frac{67}{4}\right) + \ldots..$ is :-

(1) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{2}\right)$ (2) $\frac{\pi}{2} - \cot^{-1}\left(\frac{1}{2}\right)$

(3) $\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{2}\right)$ (4) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$

Ans. **(4)**

Sol. $T_n = \tan^{-1}\left(\frac{4}{4n^2 + 3}\right)$

$T_n = \tan^{-1}\left(\frac{\left(n + \frac{1}{2}\right) - \left(n - \frac{1}{2}\right)}{1 + \left(n + \frac{1}{2}\right)\left(n - \frac{1}{2}\right)}\right)$

$T_n = \tan^{-1}\left(n + \frac{1}{2}\right) - \tan^{-1}\left(n - \frac{1}{2}\right)$

$T_1 + T_2 + \ldots. + T_n = \tan^{-1}\left(n + \frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{2}\right)$

$S_{\infty} = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$

option (4)

4. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and R be a relation on A defined by xRy if and only if $2x - y \in \{0, 1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m \;\, n$ is equal to :-

(1) 18

(2) 17

(3) 15

(4) 16

Ans. **(2)**

Sol. $2x - y = 0$

$\{0, 0\}\ \{-1, -2\}\ \{1, 2\}$

$2x - y = 1$

$\{0, -1\}\ \{1, 1\}\ \{2, 3\}\ \{-1, -3\}$

Total (0, 0) (–1, –2), (1,2) (0,-1), (1, 1) (2, 3) (–1, –3)

Reflexive $m = 5$ & $\ell = 7$

Symm. $n = 5$ $\ell + m + n = 17$

option (2)

5. Let the product of $\omega_1 = (8 + i)\sin\theta + (7 + 4i)\cos\theta$ and $\omega_2 = (1 + 8i)\sin\theta + (4 + 7i)\cos\theta$ be $\alpha + i\beta$, $i = \sqrt{-1}$. Let p and q be the maximum and the minimum values of $\alpha + \beta$ respectively.

(1) 140 (2) 130

(3) 160 (4) 150

Ans. **(2)**

Sol. $\omega_1 = (8 \sin\theta + 7 \cos\theta) + i(\sin\theta + 4 \cos\theta)$

$\omega_2 = (\sin\theta + 4 \cos\theta) + i(8 \sin\theta + 7 \cos\theta)$

$\omega_1\omega_2 = 8 \sin^2\theta + 7 \sin\theta \cos\theta + 32 \sin\theta \cos\theta + 28 \cos^2\theta - 8 \sin^2\theta - 32 \sin\theta \cos\theta - 7 \sin\theta \cos\theta - 28 \cos^2\theta + i(\sin^2\theta + 4 \sin\theta \cos\theta + 4 \sin\theta \cos\theta + 16 \cos^2\theta + 64 \sin^2\theta + 56 \sin\theta \cos\theta + 56 \sin\theta \cos\theta + 49 \cos^2\theta)$

$\omega_1\omega_2 = 0 + i(65 \sin^2\theta + 120 \sin\theta \cos\theta + 65 \cos^2\theta)$

$\alpha + \beta = 65 + 60 \sin 2q$

$\alpha + \beta|_{max} = 125$

$\alpha + \beta|_{min} = 5$

Ans. = 125 + 5 = 130

option (2)

6. Let the values of p, for which the shortest distance between the lines $\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$ and $\vec{r} = (p\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ is $\frac{1}{\sqrt{6}}$, be a, b, (a < b). Then the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :-

(1) 9

(2) $\frac{3}{2}$

(3) $\frac{2}{3}$

(4) 18

Ans. **(3)**

Sol. shortest distance $= \frac{|(\bar{a} - \bar{b})|.(\bar{p} \times \bar{q})}{|\bar{p} \times \bar{q}|}$

where

$\bar{a} = -\hat{i} + 0\hat{j} + 0\hat{k}$

$> \bar{a} - \bar{b} = (-1-p)\hat{i} - 2\hat{j} - \hat{k}$

$\bar{b} = p\hat{i} + 2\hat{j} + \hat{k}$

$\bar{p} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\bar{q} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\frac{1}{16} = \frac{|-1-p+4-1|}{\sqrt{6}}$

$|-p + 2| = 1$

$p = 3$ & $q = 1$

$\frac{x^2}{1^2} + \frac{y^2}{3^3} = 1$

$L.R = \frac{2a^2}{b} = \frac{2 \times 1}{3} = \frac{2}{3}$

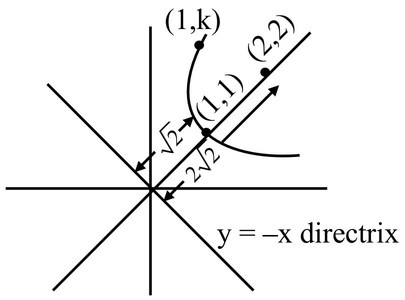
option (3)

7. The axis of a parabola is the line y = x and its vertex and focus are in the first quadrant at distances $\sqrt{2}$ and $2\sqrt{2}$ units from the origin, respectively. If the point (1, k) lies on the parabola, then a possible value of k is :-

(1) 4 (2) 9
(3) 3 (4) 8

Ans. **(2)**

Sol.



Directrix x + y = 0

PS = PM

$$\sqrt{(1-2)^2+(K-2)^2} = \frac{(1+K)}{\sqrt{2}}$$

$2K^2 + 8 - 8K + 2 = K^2 + 1 + 2K$

$K^2 - 10K + 9 = 0$

$K = 9$

option (2)

8. Let the domains of the functions

$f(x) = \log_4\log_3\log_7(8 - \log_2(x^2 + 4x + 5))$ and

$g(x) = \sin^{-1}\left(\frac{7x+10}{x-2}\right)$ be (α, β) and $[\gamma, \delta]$, respectively. Then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is equal to :-

(1) 15 (2) 13
(3) 16 (4) 14

Ans. **(1)**

Sol. $\log_3(\log_7(8 - \log_2(x^2 + 4x + 5))) > 0$

$\log_2(x^2 + 4x + 5) < 1$

$x^2 + 4x + 3 < 0$

$\Rightarrow x \in (-3, -1)$

$$-1 \le \frac{7x+10}{x-2} \le 1$$

$\Rightarrow x \in [-2, -1]$

$\alpha = -3, \beta = -1, \gamma = -2, \delta = -1$

$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 15$

option (1)

9. A line passing through the point A(–2, 0), touches the parabola P : $y^2 = x - 2$ at the point B in the first quadrant. The area, of the region bounded by the line AB, parabola P and the x-axis, is :-

(1) $\frac{7}{3}$ (2) 2

(3) $\frac{8}{3}$ (4) 3

Ans. **(3)**

Sol. Tangent

$y = m(x + 2)$

$y^2 = x - 2$

$(m(n + 2))^2 = n - 2$

$m^2x^2 + (4m^2-1)x + (4m^2 + 2) = 0$

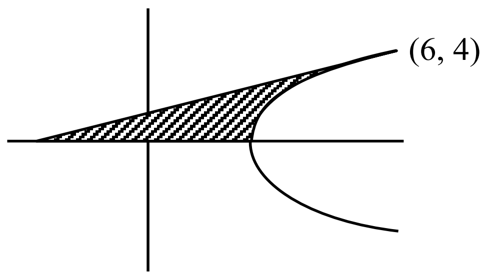
$D = 0$

$(4m^2-1)^2 - 4m^2(4m^2 + 2) = 0$

$m = \frac{1}{4}$

$y = \frac{1}{4}(n+2)$

and point of tangency (6, 2)



$$\text{Area } A = \int_0^2 \left((y^2+2)-(4y-2)\right)dy$$

$A = \frac{8}{3}$

option (3)

10. Let the sum of the focal distances of the point P(4, 3) on the hyperbola H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $8\sqrt{\frac{5}{3}}$. If for H, the length of the latus rectum is l and the product of the focal distances of the point P is m, then $9l^2 + 6$ m is equal to :-

(1) 184 (2) 186

(3) 185 (4) 187

Ans. **(3)**

Sol. $ex + a + ex - a = 8\sqrt{\frac{5}{3}}$

$2ex = 8\sqrt{\frac{5}{3}}$

$2e \times 4 = 8\sqrt{\frac{5}{3}}$

$e = \sqrt{\frac{5}{3}}$

$b^2 = a^2\left(\left(\frac{\sqrt{5}}{3}\right)^2 - 1\right)$

$b^2 = \frac{2}{3}a^2$

$\frac{16}{a^2}-\frac{9}{b^2}=1$

and $b^2 = \frac{2}{3}a^2$

$\Rightarrow a^2 = \frac{5}{2} \quad b^2 = \frac{5}{3}$

Now,

$\ell = \frac{2b^2}{a}$

$\ell^2 = \frac{4b^4}{a^2}$

$9\ell^2 = 36 \times \frac{25}{9\times 5} \times 2$

$9\ell^2 = 40$

m = (ex + a) (ex – a)

$m = e^2x^2 - a^2$

$= \frac{5}{3}\times 16 - \frac{5}{2} = \frac{145}{6}$

= 6m = 145

$9\ell^2 + 6$ m

40 + 145 = 185

option (3)

11. Let the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfy $A^n = A^{n-2} + A^2 -$ I for $n \geq 3$. Then the sum of all the elements of A^{50} is :-

(1) 53 (2) 52

(3) 39 (4) 44

Ans. **(1)**

Sol. $A^{50} = A^{48} + A^2 - I$

$= A^{46} + 2(A^2 - I)$

$= A^{44} + 3(A^2 - I)$

$= A^2 + 24\,(A^2 - I)$

$= 25A^2 - 24I$

$= 25\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} - 24\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$

Sum = 53

option (1)

12. If the sum of the first 20 terms of the series

$$\frac{4.1}{4+3.1^2+1^4}+\frac{4.2}{4+3.2^2+2^4}+\frac{4.3}{4+3.3^2+3^4}+\frac{4.4}{4+3.4^2+4^4}+....$$

is $\frac{m}{n}$, where m and n are coprime, then m + n is equal to :-

(1) 423 (2) 420

(3) 421 (4) 422

Ans. **(3)**

Sol. $\sum_{r=1}^{20} \frac{4r}{4+3r^2+r^4}$

$\sum_{r=1}^{20} \frac{4r}{(r^2+r+2)(r^2-r+2)}$

$2\sum_{r=1}^{20}\left(\frac{1}{r^2-r+2}-\frac{1}{r^2+r+2}\right)$

$2\left(\frac{1}{2}-\frac{1}{4}\right)$

$\frac{1}{4}-\frac{1}{8}$

$\frac{1}{8}-\frac{1}{14}$

$\left(\frac{1}{382}-\frac{1}{422}\right)$

$=2\left(\frac{1}{2}-\frac{1}{422}\right)$

$=\frac{420}{422}$

$=\frac{210}{211}$

option (3)

13. If

$1^2.\left({}^{15}C_1\right)+2^2.\left({}^{15}C_2\right)+3^2.\left({}^{15}C_3\right)+....+15^2.\left({}^{15}C_{15}\right)=$ $2^m\cdot3^n\cdot5^k$, where m, n, k $\in$ N, then m + n + k is equal to :-

(1) 19 (2) 21

(3) 18 (4) 20

Ans. **(1)**

Sol. $\sum_{r=1}^{15} r^2\left({}^{15}C_r\right) \Rightarrow 15\sum_{r=1}^{15} r\,{}^{14}C_{r-1}$

$15\sum_{r=1}^{15}(r-1+1)\,{}^{14}C_{r-1}$

$15\cdot14\sum_{r=1}^{15}{}^{13}C_{r-2}+15\sum_{r=1}^{15}{}^{14}C_{r-1}$

$15\cdot14\cdot2^{13}+15\cdot2^{14}$

$3^1\cdot2^{13}(70+10)$

$3^1\cdot2^{13}\cdot80$

$3^1\cdot5^1\cdot2^{17}$

m = 17 n = 1 k = 1

option (1)

14. Let for two distinct values of p the lines y = x + p touch the ellipse E : $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$ at the points A and B. Let the line y = x intersect E at the points C and D. Then the area of the quadrilateral ABCD is equal to

(1) 36 (2) 24

(3) 48 (4) 20

Ans. **(2)**

Sol. Point of contact are $\left(\frac{\mp a^2m}{\sqrt{a^2m^2+b^2}},\frac{\pm b^2}{\sqrt{a^2m^2+b^2}}\right)$

$A\left(\frac{-16}{5},\frac{9}{5}\right)B\left(\frac{16}{5},\frac{-9}{5}\right)$

Point D is $\left(\frac{12}{5},\frac{12}{5}\right)$

Area of ABD = $\frac{1}{2}\begin{vmatrix}-\frac{16}{5} & \frac{9}{5} & 1\\ \frac{16}{5} & \frac{-9}{5} & 1\\ \frac{12}{5} & \frac{12}{5} & 1\end{vmatrix}$

= 12

Area of ABCD is = 24

option (2)

15. Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of AP's in A and B respectively such that D = d + 3, d > 0. If $\frac{p+q}{p-q}=\frac{19}{5}$, then p – q is equal to

(1) 600 (2) 450

(3) 630 (4) 540

Ans. **(4)**

Sol. Let A(a – d, a, a + d) B (b – D, b, b + D)

a = 12 b = 12

$p = 12(144-d^2)$

$q = 12(144-D^2)$

$$\frac{p+q}{p-q}=\frac{19}{5}$$

$$\frac{p}{q}=\frac{24}{14}=\frac{12}{7}$$

$$\frac{144-d^2}{144-\left(d^2+6d+9\right)}=\frac{12}{7}$$

$1008 - 7d^2 = - 12d^2 - 72d + 1620$

$5d^2 + 72d - 612 = 0$

$d = 6$

$D = 9$

$p - q = 12\,(D^2 - d^3)$

$= 12(81 - 36)$

$= 12(45)$

$= 540$

option (4)

16. If a curve $y = y(x)$ passes through the point $\left(1, \frac{\pi}{2}\right)$ and satisfies the differential equation $(7x^4 \cot y - e^x \operatorname{cosec} y)\frac{dx}{dy} = x^5,\ x \geq 1$, then at $x = 2$, the value of cosy is:

(1) $\frac{2e^2-e}{64}$ (2) $\frac{2e^2+e}{64}$

(3) $\frac{2e^2-e}{128}$ (4) $\frac{2e^2+e}{128}$

Ans. **(3)**

Sol. $\frac{dy}{dx}=\frac{7\cot y}{x}-\frac{e^x \operatorname{cosec} y}{x^5}$

$$\frac{dy}{dx}=\frac{7\cot y}{\sin y.x}-\frac{e^x}{\sin y x^5}$$

$$\sin y\frac{dy}{dx}-\cos y.\frac{7}{x}=\frac{-e^x}{x^5}$$

let $-\cos y = t$

$$\sin y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}+\frac{7t}{x}=\frac{-e^x}{x^5}$$

I.F. $= x^7$

$t.\, x^7 = -\int x^2e^x dx$

$\cos y\, x^7 = x^2e^x - 2\int xe^x dx$

$\cos y\, x^7 = x^2e^x - 2xe^x + 2e^x + c$

$x = 1,\ y = \frac{\pi}{2},\ c = -e$

$$\cos y=\frac{2e^2-e}{128}$$

option (3)

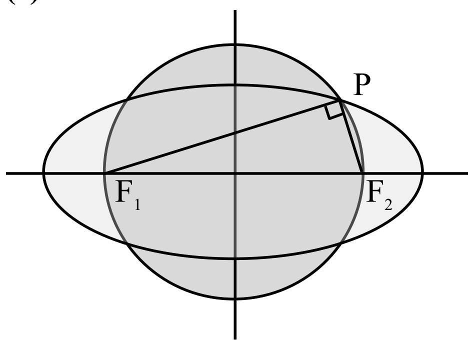
17. The centre of a circle C is at the centre of the ellipse $E : \frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, $a > b$. Let C pass through the foci F_1 and F_2 of E such that the circle C and the ellipse E intersect at four points. Let P be one of these four points. If the area of the triangle PF_1F_2 is 30 and the length of the major axis of E is 17, then the distance between the foci of E is :

(1)26 (2) 13

(3)12 (4) $\frac{13}{2}$

Ans. **(2)**

Sol.



$\frac{1}{2}PF_1.PF_2 = 30$

$PF_1 + PF_2 = 17$

$PF_1 = 12\ \ PF_2 = 5$

$F_1 F_2 = 13$

option (2)

18. Let $f(x) + 2f\left(\frac{1}{x}\right) = x^2 + 5$ and

$2g(x) - 3g\left(\frac{1}{2}\right) = x,\ x > 0$. If $\alpha = \int_1^2 f(x)\,dx$, and

$\beta = \int_1^2 g(x)\,dx$, then the value of $9\alpha + \beta$ is :

(1) 1 (2) 0

(3) 10 (4) 11

Ans. (4)

Sol. $f(x) + 2f\left(\frac{1}{x}\right) = x^2 + 5$

$$f\left(\frac{1}{x}\right) + 2f(x) = \frac{1}{x^2} + 5$$

$$f(x) = \frac{2}{3x^2} - \frac{x^2}{3} + \frac{5}{3}$$

$$\alpha = \int_1^2 \left(\frac{2}{3x^2} - \frac{x^2}{3} + \frac{5}{3}\right)dx$$

$$\left(-\frac{2}{3x} - \frac{x^3}{9} + \frac{5x}{3}\right)_1^2$$

$$-\frac{1}{3} - \frac{8}{9} + \frac{10}{3} + \frac{2}{3} + \frac{1}{9} - \frac{5}{3}$$

$$\alpha = 2 - \frac{7}{9} = \frac{11}{9}$$

$$2g(x) - 3g\left(\frac{1}{2}\right) = x$$

$$g\left(\frac{1}{2}\right) = -\frac{1}{2}$$

$$g(x) = \frac{x}{2} - \frac{3}{4}$$

$$\beta = \int_1^2 \left(\frac{x}{2} - \frac{3}{4}\right)dx$$

$$\left(\frac{x^2}{4} - \frac{3x}{4}\right)_1^2 = 1 - \frac{3}{2} - \frac{1}{4} + \frac{3}{4} = 0$$

$9\alpha + \beta = 11$

option (4)

19. Let A be the point of intersection of the lines

$L_1 : \frac{x-7}{1} = \frac{y-5}{0} = \frac{z-3}{-1}$ and

$L_2 : \frac{x-1}{3} = \frac{y+3}{4} = \frac{z+7}{5}$. Let B and C be the point on the lines L_1 and L_2 respectively such that $AB = AC = \sqrt{15}$. Then the square of the area of the triangle ABC is :

(1) 54 (2) 63

(3) 57 (4) 60

Ans. (1)

Sol. Angle between both lines

$$\cos\theta = \left|\frac{3+0-5}{\sqrt{2}\sqrt{50}}\right|$$

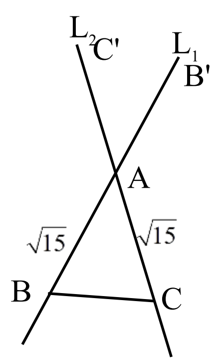
$$\sin\theta = \frac{2}{10} = \frac{1}{5}$$

$$\sin\theta = \frac{\sqrt{24}}{5}$$

$$\text{area} = \frac{1}{2}ab\sin\theta$$

$$\frac{1}{2}\sqrt{15}\sqrt{15}\frac{\sqrt{24}}{5}$$

square of area $\frac{15\,.\,15\,.\,24}{4\,.\,25}$



option (1)

20. Let the mean and the standard deviation of the observation 2, 3, 3, 4, 5, 7, a, b be 4 and $\sqrt{2}$ respectively. Then the mean deviation about the mode of these observations is :

(1) 1 (2) $\frac{3}{4}$

(3) 2 (4) $\frac{1}{2}$

Ans. **(1)**

Sol. $\frac{24+a+b}{8}=4$

$a + b = 8$

$2 = \frac{4+1+1+0+1+9+(a-4)^2+(b-4)^2}{8}$

$16 = 48 + a^2 + b^2 - 8a - 8b$

$a^2 + b^2 = 32$

$32 = 2ab$

$ab = 16$

$a = 4 \quad b = 4$

mode = 4

mean deviation $= \frac{2+1+1+0+1+3+0+0}{8} = 1$

option (1)

SECTION-B

21. If α is a root of the equation $x^2 + x + 1 = 0$ and $\sum_{k=1}^{n}\left(\alpha^k + \frac{1}{\alpha^k}\right)^2 = 20$, then n is equal to ______

Ans. **(11)**

Sol. $\alpha = \omega$

$\therefore \left(\omega^k + \frac{1}{\omega^k}\right)^2 = \omega^{2k} + \frac{1}{\omega^{2k}} + 2$

$= \omega^{2k} + \omega^k + 2 \qquad \because \omega^{3k} = 1$

$\therefore \sum_{k=1}^{n}(\omega^{2k} + \omega^k + 2) = 20$

$\Rightarrow (\omega^2 + \omega^4 + \omega^6 + \ldots + \omega^{2n}) + (\omega + \omega^2 + \omega^3 + \ldots + \omega^n) + 2n = 20$

Now if $n = 3m, \quad m \in I$

Then $0 + 0 + 2n = 20 \Rightarrow n = 10$ (not satisfy)

if $n = 3m+1$, then

$\omega^2 + \omega + 2n = 20$

$-1 + 2n = 20 \Rightarrow n = \frac{21}{2}$ (not possible)

if $n = 3m+2$,

$(\omega^8 + \omega^{10}) + (\omega^4 + \omega^5) + 2n = 20$

$\Rightarrow (\omega^2 + \omega) + (\omega + \omega^2) + 2n = 20$

$2n = 22$

$n = 11$ satisfy $n = 3m + 2$

$\therefore n = 11$

22. If $\int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9}dx = \frac{1}{m}\left(\left(\sqrt{1+x^2}+x\right)^n\left(n\sqrt{1+x^2}-x\right)\right) + C$ where C is the constant of integration and m, n ∈ N, then m + n is equal to

Ans. **(379)**

Sol. rationalise

$\Rightarrow \int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9} \times \frac{\left(\sqrt{1+x^2}+x\right)^9}{\left(\sqrt{1+x^2}+x\right)^9}dx$

$\Rightarrow \int \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{1}dx$

Put $\sqrt{1+x^2} + x = t$

$\left(\frac{x}{\sqrt{1+x^2}}+1\right)dx = dt$

$$dx = \frac{dt}{t}\sqrt{1+x^2}$$

Now as $\sqrt{1+x^2} + x = t$

so $\sqrt{1+x^2} - x = \frac{1}{t}$

$\therefore \sqrt{1+x^2} = \frac{1}{2}\left(t+\frac{1}{t}\right)$

Thus $I = \int t^{19}.\frac{dt}{t}.\frac{1}{2}\left(t+\frac{1}{t}\right)$

$\Rightarrow \frac{1}{2}\int\left(t^{19}+t^{17}\right)dt$

$= \frac{1}{2}\left(\frac{t^{20}}{20}+\frac{t^{18}}{18}\right)+C$

$= \frac{t^{19}}{360}\left[9t+\frac{10}{t}\right]+C$

$= \frac{t^{19}}{360}\left[9\left(t+\frac{1}{t}\right)+\frac{1}{t}\right]+C$

$\Rightarrow \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{360}\left[9\left(2\sqrt{1+x^2}\right)+\left(\sqrt{1+x^2}-x\right)\right]+C$

$\Rightarrow \frac{\left(\sqrt{1+x^2}+x\right)^{19}}{360}\left[19\sqrt{1+x^2}-x\right]+C$

$\therefore m = 360, n = 19$

$\therefore m + n = 379$

23. A card from a pack of 52 cards is lost. From the remaining 51 cards, n cards are drawn and are found to be spades. If the probability of the lost card to be a spade is $\frac{11}{50}$, the n is equal to

Ans. **(2)**

Sol. n cards are drawn & are found all spade, thus

remaining spades = 13 – x

remaining total cards = 52 – x

Now given that P(lost card is spade) = $\frac{11}{50}$

i.e. $\frac{^{13-n}C_1}{^{52-n}C_1} = \frac{11}{50}$

$50(13 - n) = 11(52 - n)$

$39n = 78$

$n = 2$

24. Let m and n, (m < n) be two 2-digit numbers. Then the total numbers of pairs (m, n), such that gcd (m, n) = 6, is _____

Ans. **(64)**

Sol. Let $m = 6a$, $n = 6b$

$m < n \Rightarrow a < b$

where a & b are co-prime numbers

also since m & n are 2 digit nos, so

$10 \le m \le 99$ & $10 \le n \le 99$

i.e. $2 \le a \le 16$ & $2 \le b \le 16$

($\because$ a is integer)

Now

$2 \le a < b \le 16$ & a & b are co-prime

$\therefore$ if

a = 2, b = 3, 5, 7, 9, 11, 13, 15

a = 3, b = 4, 5, 7, 8, 10, 11, 13, 14, 16

a = 4, b = 5, 7, 9, 11, 13, 15

a = 5, b = 6, 7, 8, 9, 11, 12, 13, 14, 16

a = 6, b = 7, 11, 13

a = 7, b = 8, 9, 10, 11, 12, 13, 15, 16

a = 8, b = 9, 11, 13, 15

a = 9, b = 10, 11, 13, 14, 16

a = 10, b = 11, 13

a = 11, b = 12, 13, 14, 15, 16

a = 12, b = 13

a = 13, b = 14, 15, 16

a = 14, b = 15

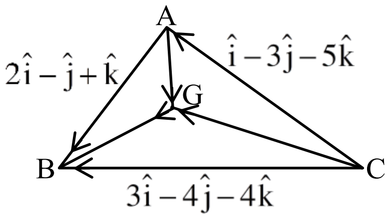
a = 15, b = 16

64 ordered pairs

25. Let the three sides of a triangle ABC be given by the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$. Let G be the centroid of the triangle ABC. Then $6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)$ is equal to ____

Ans. **(164)**

Sol.



By given data

$\overrightarrow{AB}+\overrightarrow{AC}=\overrightarrow{CB}$

Let pv of $\vec{A}$ are $\vec{O}$ then

$\overrightarrow{AB}=\vec{B}-\vec{A}$

i.e. pv of $\vec{B}-=2\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{CA}=\vec{A}-\vec{C}$

i.e. pv of $\vec{C}=-\left(\hat{i}-3\hat{j}-5\hat{k}\right)$

Now pv of centroid

$$\left(\vec{G}\right)-=\frac{\vec{A}+\vec{B}+\vec{C}}{3}=\frac{\vec{0}+(2,-1,1)+(-1,3,5)}{3}$$

$$\vec{G}=\frac{1}{3}\left(\hat{i}+2\hat{j}+6\hat{k}\right)$$

Now $\overrightarrow{AG}=\frac{1}{3}\left(\hat{i}+2\hat{j}+6\hat{k}\right)$

$$\Rightarrow |\overrightarrow{AG}|^2=\frac{1}{9}\times 41$$

$$\overrightarrow{BG}=\left(\frac{1}{3}-2\right)\hat{i}+\left(\frac{2}{3}+1\right)\hat{j}+(2-1)\hat{k}$$

$$\Rightarrow |\overrightarrow{BG}|^2=\frac{59}{9}$$

$$\overrightarrow{CG}=\left(\frac{1}{3}+1\right)\hat{i}+\left(\frac{2}{3}-3\right)\hat{j}+(2-5)\hat{k}$$

$$\Rightarrow |\overrightarrow{CG}|^2=\frac{146}{9}$$

Now

$$6\left[|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right]=6\times\left[\frac{41}{9}+\frac{59}{9}+\frac{146}{9}\right]$$

$$=6\times\frac{246}{9}=164$$

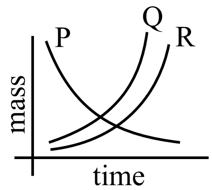
JEE–MAIN EXAMINATION – APRIL 2025

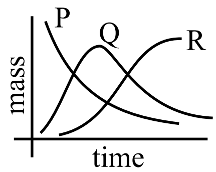
(HELD ON FRIDAY 04th APRIL 2025) **TIME : 3:00 PM TO 6:00 PM**

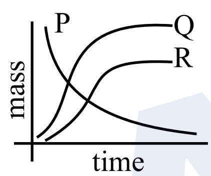
PHYSICS — TEST PAPER WITH SOLUTION

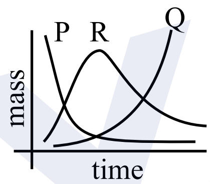
SECTION-A

26. A radioactive material P first decays into Q and then Q decays to non-radioactive material R. Which of the following figure represents time dependent mass of P, Q and R?

(1)


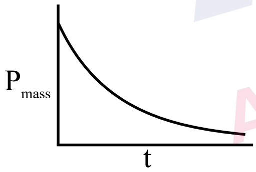
(2)


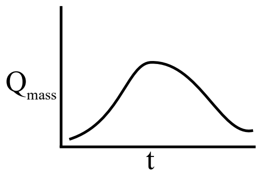
(3)


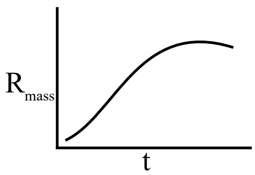
(4)


Ans. (2)

Sol. $P \rightarrow Q \rightarrow R$







27. There are 'n' number of identical electric bulbs, each is designed to draw a power p independently from the mains supply. They are now joined in series across the main supply. The total power drawn by the combination is :

(1) np (2) $\frac{p}{n^2}$

(3) $\frac{p}{n}$ (4) p

Ans. (3)

Sol. $R_s = R_1 + R_2 + R_3 + \ldots\ldots + R_n$

$$\frac{V^2}{P_s} = \frac{V^2}{P} + \frac{V^2}{P} + \ldots\ldots + \frac{V^2}{P_n}$$

$$P_s = \frac{P}{n}$$

28. Consider a rectangular sheet of solid material of length ℓ = 9 cm and width d = 4 cm. The coefficient of linear expansion is $\alpha = 3.1 \times 10^{-5}\ K^{-1}$ at room temperature and one atmospheric pressure. The mass of sheet m = 0.1 kg and the specific heat capacity $C_v = 900\ J\ kg^{-1}K^{-1}$. If the amount of heat supplied to the material is 8.1×10^2 J then change in area of the rectangular sheet is :-

(1) $2.0 \times 10^{-6}\ m^2$

(2) $3.0 \times 10^{-7}\ m^2$

(3) $6.0 \times 10^{-7}\ m^2$

(4) $4.0 \times 10^{-7}\ m^2$

Ans. (1)

Sol. $\Delta Q = ms\Delta T$

$8.1 \times 10^2 = 0.1 \times 900 \times \Delta T$

$\Delta A = A_0\, 2\, \alpha\, \Delta T = 2.0 \times 10^{-6}\ m^2$

29. Given below are two statements :

Statement (I) : The dimensions of Planck's constant and angular momentum are same.

Statement (II) : In Bohr's model electron revolve around the nucleus only in those orbits for which angular momentum is integral multiple of Planck's constant.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) Both **Statement I** and **Statement II** are correct

(2) **Statement I** is incorrect but **Statement II** is correct

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are incorrect

Ans. (3)

Sol. $E = hf$

$ML^2T^{-2} = [h] \times [T^{-1}]$

$[h] = [ML^2T^{-1}]$

$L = [MVR] = [ML^2T^{-1}]$

$$L = \frac{nh}{2\pi}$$

L is integral multiple of $\frac{h}{2\pi}$

30. A cylindrical rod of length 1 m and radius 4 cm is mounted vertically. It is subjected to a shear force of 10^5 N at the top. Considering infinitesimally small displacement in the upper edge, the angular displacement θ of the rod axis from its original position would be : (shear moduli, $G = 10^{10}$ N/m^2)

(1) $1/160\pi$

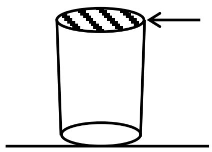
(2) $1/4\pi$

(3) $1/40\pi$

(4) $1/2\pi$

Ans. (1)

Sol.



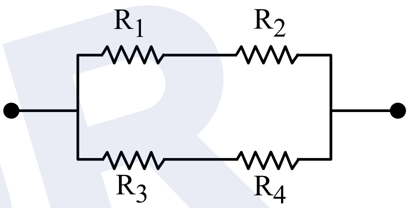
$$\text{Shear moduli} = \frac{\sigma_{shear}}{\theta}$$

$$10^{10} = \frac{10^5}{\pi \times 16 \times 10^{-4}} \times \frac{1}{\theta}$$

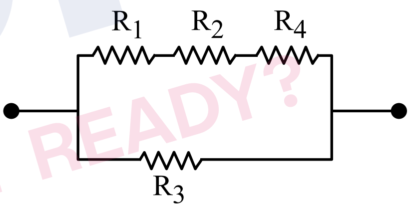
$$\theta = \frac{1}{160\pi} \text{ Radian}$$

31. From the combination of resistors with resistance values $R_1 = R_2 = R_3 = 5\ \Omega$ and $R_4 = 10\ \Omega$, which of the following combination is the best circuit to get an equivalent resistance of 6Ω ?

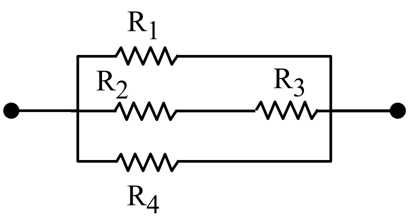
(1)



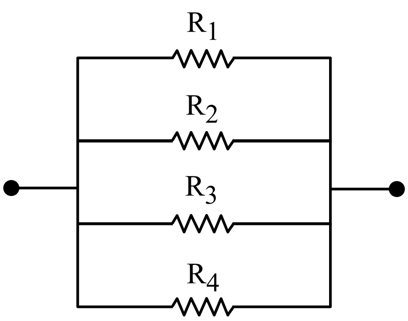
(2)



(3)

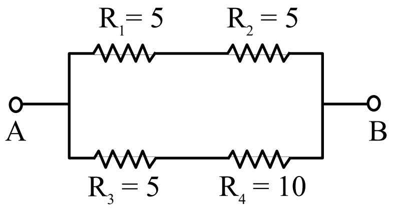


(4)

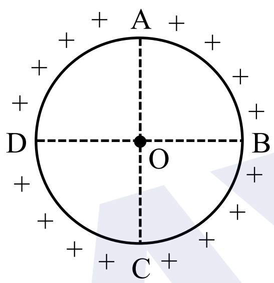


Ans. (1)

Sol. $\frac{1}{R_P} = \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{1}{6}$



32. A metallic ring is uniformly charged as shown in figure. AC and BD are two mutually perpendicular diameters. Electric field due to arc AB to 'O' is 'E' is magnitude. What would be the magnitude of electric field at 'O' due to arc ABC ?



(1) 2E

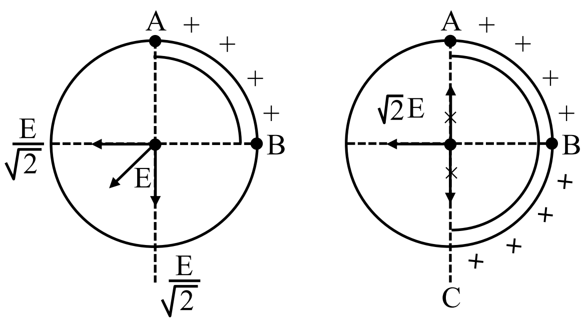
(2) $\sqrt{2}E$

(3) E/2

(4) Zero

Ans. **(2)**

Sol.

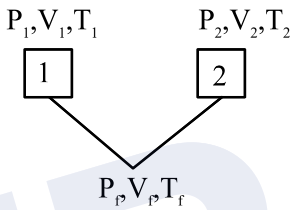


33. There are two vessels filled with an ideal gas where volume of one is double the volume of other. The large vessel contains the gas at 8 kPa at 1000 K while the smaller vessel contains the gas at 7 kPa at 500 K. If the vessels are connected to each other by a thin tube allowing the gas to flow and the temperature of both vessels is maintained at 600 K, at steady state the pressure in the vessels will be (in kPa).

(1) 4.4
(2) 6
(3) 24
(4) 18

Ans. **(2)**

Sol.



Number of masses will remain constant

$n_1 + n_2 = n_f$

$$\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{P_fV_f}{RT_f}$$

$$\frac{8 \times 2V}{R \times 1000} + \frac{7 \times V}{R \times 500} = \frac{P_f(3V)}{R \times 600}$$

$$\frac{16}{1000} + \frac{14}{1000} = \frac{P_f}{R \times 600}$$

$$\frac{30}{1000} = \frac{P_f}{200}$$

$P_f = 6$ kPa

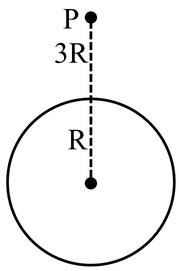
34. An object is kept at rest at a distance of 3R above the earth's surface where R is earth's radius. The minimum speed with which it must be projected so that it does not return to earth is :

(Assume M = mass of earth, G = Universal gravitational constant)

(1) $\sqrt{\frac{GM}{2R}}$
(2) $\sqrt{\frac{GM}{R}}$
(3) $\sqrt{\frac{3GM}{R}}$
(4) $\sqrt{\frac{2GM}{R}}$

Ans. **(1)**

Sol.



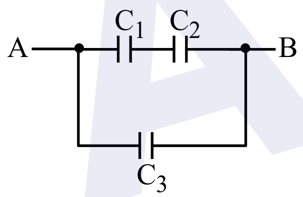
$P_P + k_P = P_O + k_0$

$$-\frac{GMm}{4R} + \frac{1}{2}mV_P^2 = 0$$

$$V_P = \sqrt{\frac{GM}{2R}}$$

Choice 1

35. Three parallel plate capacitors C_1, C_2 and C_3 each of capacitance 5 μF are connected as shown in figure. The effective capacitance between points A and B, when the space between the parallel plates of C_1 capacitor is filled with a dielectric medium having dielectric constant of 4, is :



(1) 22.5 μF (2) 7.5 μF

(3) 9 μF (4) 30 μF

Ans. **(3)**

Sol. After dielectric

$C_1 = 4C$

$C_1 = 4 \times 5 = 20\mu F$

$C_2 = C_3 = 5\mu F$

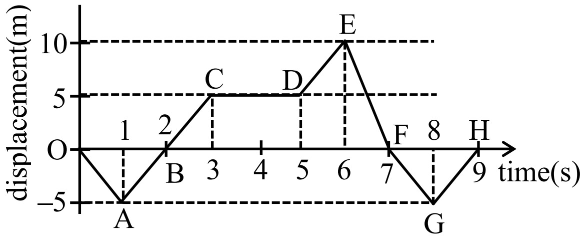
C_1 & C_2 are in series which is parallel to C_3 So

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} + C_3 \Rightarrow \frac{20\times 5}{20+5} + 5$$

$= 4 + 5 = 9$ μF

Correct Option (3)

36. The displacement x versus time graph is shown below.



(A) The average velocity during 0 to 3 s is 10 m/s

(B) The average velocity during 3 to 5 s is 0 m/s

(C) The instantaneous velocity at t = 2 s is 5 m/s

(D) The average velocity during 5 to 7 s and instantaneous velocity at t = 6.5 s are equal

(E) The average velocity from t = 0 to t = 9 s is zero

Choose the **correct** answer from the options given below:

(1) (A), (D), (E) only

(2) (B), (C), (D) only

(3) (B), (D), (E) only

(4) (B), (C), (E) only

Ans. **(4)**

Sol. $<\vec{v}> = \frac{\Delta\vec{s}}{\Delta t} = \frac{S_f - S_i}{t_f - t_i}$

$$\vec{v} = \frac{ds}{dt} = \text{slope}$$

(A) 0 to 3 sec ; $<\vec{v}> = \frac{5-0}{3} = 5/3\,m/s$

(B) 0 to 5 sec ; $<\vec{v}> = \frac{5-5}{2} = 0$

(C) t = 2; slope = $\vec{v} = 5m/s$

(D) t = 5 to 7 sec ; $<\vec{v}> = \frac{0-5}{2} = -2.5m/s$

At t= 6.5 sec; $\vec{v} = 10$

(E) t = 0 to t = 9 ; $<\vec{v}> = 0$

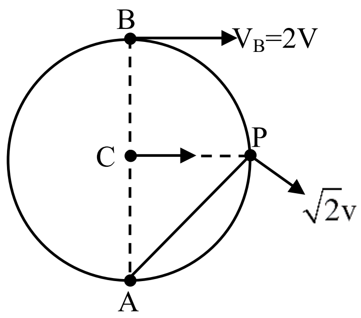
37. A wheel is rolling on a plane surface. The speed of a particle on the highest point of the rim is 8 m/s. The speed of the particle on the rim of the wheel at the same level as the centre of wheel, will be :

(1) $4\sqrt{2}$ m / s (2) 8 m/s

(3) 4 m/s (4) $8\sqrt{2}$ m / s

Ans. **(1)**

Sol.



If $V_B = 2V$

Point A is instantaneous center of rotation

Given V_B = 8 m/s

V = 4 m/s

$V_P = \sqrt{2}v \Rightarrow V_p = 4\sqrt{2}$ m / s

correct (1)

38. For the determination of refractive index of glass slab, a travelling microscope is used whose main scale contains 300 equal divisions equals to 15 cm. The vernier scale attached to the microscope has 25 divisions equals to 24 divisions of main scale. The least count (LC) of the travelling microscope is (in cm) :

(1) 0.001 (2) 0.002

(3) 0.0005 (4) 0.0025

Ans. **(2)**

Sol. 300 msd = 15 cm

$$1 \text{ msd} = \frac{15}{300} \text{cm} = 0.05\text{cm}$$

25 vsd = 24 msd

$$1 \text{ vsd} = \frac{24}{25} \text{msd}$$

LC = 1 msd – 1 vsd

$$LC = 1 \text{ msd} - \frac{24}{25} \text{msd} = \frac{1}{25} \text{msd}$$

$$LC = \frac{1}{25} \times 0.05 = 0.002 \text{ cm}$$

correct option (2)

39. A block of mass 25 kg is pulled along a horizontal surface by a force at an angle 45° with the horizontal. The friction coefficient between the block and the surface is 0.25. The displacement of 5 m of the block is:

(1) 970 J

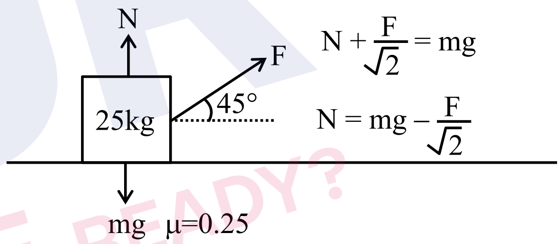
(2) 735 J

(3) 245 J

(4) 490 J

Ans. **(3)**

Sol.



$$N + \frac{F}{\sqrt{2}} = mg$$

$$N = mg - \frac{F}{\sqrt{2}}$$

Block travels with uniform velocity

So a = 0 ⇒ F cos 45° = friction

$$\frac{F}{\sqrt{2}} = \mu\left[mg - \frac{F}{\sqrt{2}}\right]$$

$$\frac{F}{\sqrt{2}} = 0.25\left[25 \times 9.8 - \frac{F}{\sqrt{2}}\right]$$

$$\Rightarrow 1.25 \frac{F}{\sqrt{2}} = 61.25$$

$$F = \frac{61.25 \times \sqrt{2}}{1.25} = 49\sqrt{2}$$

W_{ext} = FS cos 45°

$$= 49\sqrt{2} \times 5 \times \frac{1}{\sqrt{2}} = 245 \text{ J}$$

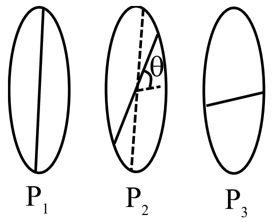
40. Two polarisers P_1 and P_2 are placed in such a way that the intensity of the transmitted light will be zero. A third polariser P_3 is inserted in between P_1 and P_2, at the particular angle between P_2 and P_3. The transmitted intensity of the light passing the through all three polarisers is maximum. The angle between the polarisers P_2 and P_3 is :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{8}$ (4) $\frac{\pi}{3}$

Ans. **(1)**

Sol. Through P_2 $I_1 = I_0 \sin^2\left(\frac{\pi}{2}-\theta\right)$



$I_1 = I_0 \cos^2\theta$

Through P_3 $I_{net} = \left(I_0 \cos^2\theta\right)\sin^2\theta$

$I_{net} = \frac{I_0}{4}[\sin(2\theta)]^2$ for max I_{net} $\theta = 45^o$

So angle between P_2 and P_3 = $\frac{\pi}{4}$

Correct Ans. (1)

41. Consider a n-type semiconductor in which n_e and n_h are number of electrons and holes, respectively.

(A) Holes are minority carriers

(B) The dopant is a pentavalent atom

(C) $n_e n_h \neq n_i^2$

(where n_i is number of electrons or holes in semiconductor when it is intrinsic form)

(D) $n_e n_h \geq n_i^2$

(E) The holes are not generated due to the donors

Choose the **correct** answer from the options given below :

(1) (A), (C), (D) only (2) (A), (C), (E) only

(3) (A), (B), (E) only (4) (A), (B), (C) only

Ans. **(3)**

Sol. (A) n type semiconductor holes are minority carriers and e^- are majority carriers

(B) Dopant are pentavalent atom.

(C) $n_e \cdot n_h = n_i^2$ for intrinsic semiconductor

(E) In n type semiconductor primary source of holes generation are thermal excitation.

42. Match **List-I** with **List-II**.

List-I	**List-II**
(A) Isobaric	(I) $\Delta Q = \Delta W$
(B) Isochoric	(II) $\Delta Q = \Delta U$
(C) Adiabatic	(III) ΔQ = zero
(D) Isothermal	(IV) $\Delta Q = \Delta U + P\Delta V$

ΔQ = Heat supplied

ΔW = Work done by the system

ΔU = Change in internal energy

P = Pressure of the system

ΔV = Change in volume of the system

Choose the **correct** answer from the options given below :

(1) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(2) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(4) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

Ans. **(3)**

Sol. (A) Isobaric (P = C)

$\boxed{\Delta Q = \Delta U + P\Delta V}$

(B) Isochoric (V = C)

$\boxed{\Delta Q = \Delta U}$

(C) Adiabatic ($\Delta Q = 0$)

$\boxed{\Delta Q = 0}$

(D) Isothermal ($\Delta U = 0$)

$\boxed{\Delta Q = \Delta W}$

43. Displacement of a wave is expressed as $x(t) = 5\cos\left(628t + \frac{\pi}{2}\right)m$. The wavelength of the wave when its velocity is 300 m/s is :

(1) 5 m (2) 3 m

(3) 0.5 m (4) 0.33 m

Ans. (2)

Sol. $x(t) = 5\cos\left[628t + \frac{\pi}{2}\right]m$

velocity (v_ω) = 300 m/s

$$v_w = \frac{\omega}{K}$$

$$300 = \frac{628}{K} \Rightarrow K = \frac{628}{300}$$

$$\frac{2\pi}{\lambda} = \frac{628}{300} \Rightarrow \lambda = \frac{2\times3.14\times300}{628} \quad \lambda = 2m$$

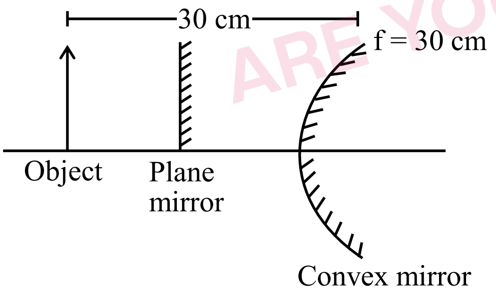
44. A finite size object is placed normal to the principal axis at a distance of 30 cm from a convex mirror of focal length 30 cm. A plane mirror is now placed in such a way that the image produced by both the mirrors coincide with each other. The distance between the two mirrors is :

(1) 45 cm (2) 7.5 cm

(3) 22.5 cm (4) 15 cm

Ans. (2)

Sol.



For Convex mirror

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{30} = \frac{1}{30}$$

$$\frac{1}{v} = \frac{2}{30} = \frac{1}{15} \Rightarrow v = 15\text{ cm}$$

Image formed by convex mirror is at 45cm from object so plane mirror should be placed midway at 22.5 cm from object so that both of their images may coinside,

Therefore distance between both mirrors

= 30 – 22.5 = 7.5 cm

Correct Answer : Option 2

45. In an electromagnetic system, a quantity defined as the ratio of electric dipole moment and magnetic dipole moment has dimension of $[M^P L^Q T^R A^S]$. The value of P and Q are :

(1) – 1, 0 (2) – 1, 1

(3) 1, – 1 (4) 0, – 1

Ans. (4)

Sol. Electric dipole moment $(\vec{P}) = q \times 2\ell$

Magnetic dipole moment $(\vec{M}) = IA$

$$\left[\frac{P}{M}\right] = \left[\frac{LTA}{L^2A}\right] = L^{-1}T = M^0L^{-1}T^1A^0$$

After compering values of P & Q are 0, –1

Correct Answer : Option 4

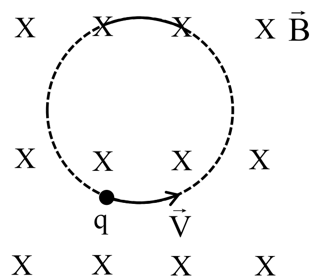
SECTION-B

46. A particle of charge 1.6 μC and mass 16 μg is present in a strong magnetic field of 6.28 T. The particle is then fired perpendicular to magnetic field. The time required for the particle to return to original location for the first time is ________ s. ($\pi = 3.14$)

Allen Ans. 0

NTA Ans. 10

Sol.



Angle between $\vec{V}$ of charge & $\vec{B}$ is 90° motion will be uniform circular motion time period is given by

$$T = \frac{2\pi m}{qB} = \frac{2\pi\times16\times10^{-9}\,kg}{1.6\times10^{-6}\times6.28}$$

T = 0.01 seconds

NTA Answer is 10

Correct Answer is 0 (nearest integer)

47. A solid sphere with uniform density and radius R is rotating initially with constant angular velocity (ω_1) about its diameter. After some time during the rotation its starts loosing mass at a uniform rate, with no change in its shape. The angular velocity of the sphere when its radius become R/2 is $x\omega_1$. The value of x is ______.

Ans. **(32)**

Sol. When sphere is of radius R, its mass is M, when radius is reduced to $\frac{R}{2}$, mass will reduced to $\frac{M}{8}$

Now by conservation of angular momentum ($\tau_{ext} = 0$)

$L_1 = L_2$

$I_1\omega_1 = I_2\omega_2$

$$\left(\frac{2}{5}MR^2\right)\omega_1 = \left(\frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R}{2}\right)^2\right)\omega_2$$

$\boxed{\omega_2 = 32\,\omega_1}$ value of x is 32

Answer is 32

48. If an optical medium possesses a relative permeability of $\frac{10}{\pi}$ and relative permittivity of $\frac{1}{0.0885}$, then the velocity of light is greater in vacuum than that in this medium by ______ times. ($\mu_0 = 4\pi \times 10^{-7}$ H/m, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m, $c = 3 \times 10^8$ m/s)

Ans. **(6)**

Sol. Since velocity of light in terms of μ & E is

$$V = \frac{1}{\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_0\mu_r}} \times \frac{1}{\sqrt{\in_0 \in_r}}$$

$$= \frac{1}{\sqrt{\mu_r \in_r}} \times \frac{1}{\sqrt{\mu_0 \in_0}}$$

$$= \frac{C}{\sqrt{\mu_r \in_r}} = \frac{C}{\sqrt{\frac{10}{\pi} \times \frac{1}{0.0885}}}$$

$$= \frac{C}{\sqrt{36}} = \frac{C}{6}$$

$$V = \frac{C}{6}$$

$C = 6V$

Velocity of light in vacuum is greater by 6 times the velocity of light in medium

Answer is 6

49. In a Young's double slit experiment, two slits are located 1.5 mm apart. The distance of screen from slits is 2 m and the wavelength of the source is 400 nm. If the 20 maxima of the double slit pattern are contained within the centre maximum of the single slit diffraction pattern, then the width of each slit is $x \times 10^{-3}$ cm, where x-value is _____.

Ans. **(15)**

Sol. Width of 20 maxima of double slit = width of central maxima of single slit

$$\frac{20\lambda D}{d} = \frac{2\lambda D}{a}$$

$$\frac{10}{d} = \frac{1}{a}$$

$$a = \frac{d}{10} = \frac{1.5\times10^{-1}}{10}\text{cm} = 15\times10^{-3}\text{cm}$$

Value of x is 15

Answer is 15

50. An inductor of self inductance 1 H connected in series with a resistor of 100 π ohm and an ac supply of 100 π volt, 50 Hz. Maximum current flowing in the circuit is ______ A.

Ans. **(1)**

Sol. Impedance of circuit

$$Z = \sqrt{R^2 + (X_L)^2} = \sqrt{R^2 + (\omega_L)^2}$$

$$= \sqrt{(100\pi)^2 + (2\pi\times50\times1)^2}$$

$$= \sqrt{(100\pi)^2 + (100\pi)^2}$$

$$= \sqrt{2}\times100\pi$$

$$I_{rms} = \frac{V}{2} = \frac{100\pi}{\sqrt{2}\times100\pi} = \frac{1}{\sqrt{2}}$$

$$I_{max} = \sqrt{2}\,I_{rms} = \sqrt{2}\times\frac{1}{\sqrt{2}} = 1\,\text{Ampere}$$

Correct Answer : 1

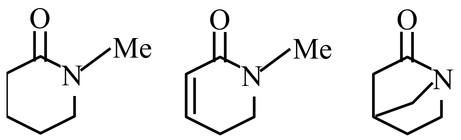
JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON FRIDAY 04th APRIL 2025) **TIME : 3:00 PM TO 6:00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

51. The correct order of basicity for the following molecules is :

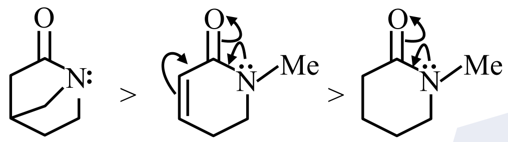


(P) (Q) (R)

(1) P > Q > R (2) R > P > Q
(3) Q > P > R (4) R > Q > P

Ans. (4)

Sol.



(R) According to Bredt's rule it is localised lone pair. > (Q) cross conjugation > (P)

52. The incorrect relationship in the following pairs in relation to ionisation enthalpies is :

(1) $Mn^{+} < Cr^{+}$ (2) $Mn^{+} < Mn^{2+}$
(3) $Fe^{2+} < Fe^{3+}$ (4) $Mn^{2+} < Fe^{2+}$

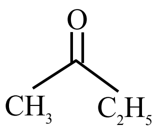
Ans. (4)

Sol. Mn^{2+} : $[Ar]3d^5$
Half filled stability
More IE than Fe^{2+}
Fe^{2+} : $[Ar]3d^6$

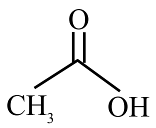
53. Which among the following compounds give yellow solid when reacted with NaOI/NaOH?

(A) $CH_3-CH(OH)-C_2H_5$ (B) $CH_3-CH_2-CH_2-OH$

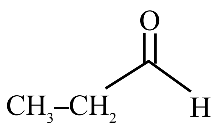
(C)



(D)



(E)



Choose the **correct** answer from the options given below :

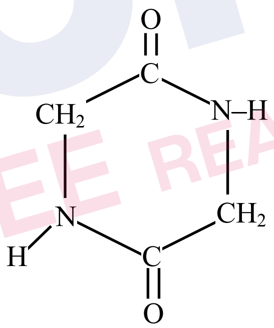
(1) (B), (C) and (E) Only
(2) (A) and (C) Only
(3) (C) and (D) Only
(4) (A), (C) and (D) Only

Ans. (2)

Sol. $CH_3 + CH(OH) - CH_2 - CH_3 \xrightarrow[NaOH]{NaOI} \underset{\text{Yellow solid}}{CHI_3} + CH_3 - CH_2 - C(=O) - \bar{O}\overset{+}{Na}$

$CH_3 - C(=O) - CH_2 - CH_3 \xrightarrow[NaOH]{NaOI} \underset{\text{Yellow solid}}{CHI_3} + CH_3 - CH_2 - C(=O) - \bar{O}\overset{+}{Na}$

54. A dipeptide, "x" on complete hydrolysis gives "y" and "z". "y" on treatment with aq. HNO_2 produces lactic acid. On the other hand "z" on heating gives the following cyclic molecule.



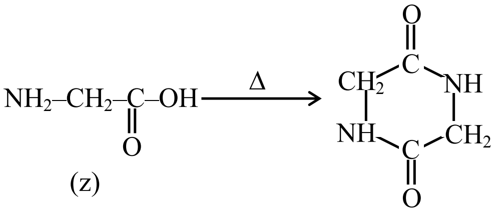
Based on the information given, the dipeptide X is:

(1) valine-glycine (2) alanine-glycine
(3) valine-leucine (4) alanine-alanine

Ans. (2)

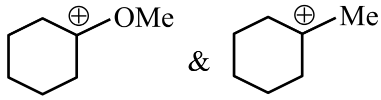
Sol. $\underset{\text{(alanine-glycine) (x)}}{NH_2-CH(CH_3)-C(=O)+NH-CH_2-C(=O)-OH} \xrightarrow{\text{Hydrolysis}} NH_2-CH(CH_3)-C(=O)-OH \ (y) + NH_2-CH_2-C(=O)-OH \ (z)$

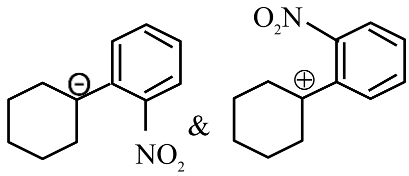
$\underset{(y)}{NH_2-CH(CH_3)-C(=O)-OH} \xrightarrow{aq.\ HNO_2} \underset{\text{Lactic acid}}{HO-CH(CH_3)-C(=O)-OH}$

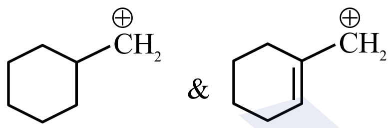


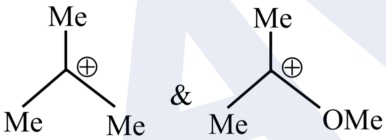
(z)

55. In which pairs, the first ion is more stable than the second ?

(A) 

(B) 

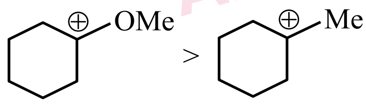
(C) 

(D) 

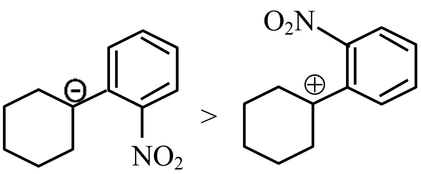
(1) (B) & (D) only (2) (A) & (B) only

(3) (B) & (C) only (4) (A) & (C) only

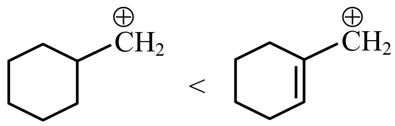
Ans. (2)

Sol. (A) 

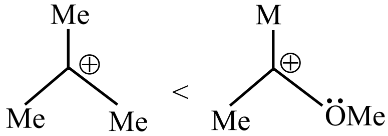
Stable by back bonding

(B) 

Stable by –M, –I effect of NO_2 group

(C) 

Stable by resonance

(D) 

Stable by back bonding

56. Given below are two statements :

Statement (I) : Alcohols are formed when alkyl chlorides are treated with aqueous potassium hydroxide by elimination reaction.

Statement (II) : In alcoholic potassium hydroxide, alkyl chlorides form alkenes by abstracting the hydrogen from the β-carbon.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) Both **Statement I** and **Statement II** are incorrect

(2) **Statement I** is incorrect but **Statement II** is correct

(3) **Statement I** is correct but **Statement II** is incorrect

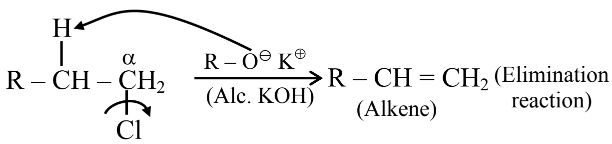
(4) Both **Statement I** and **Statement II** are correct

Ans. (2)

Sol. Statement (I) :

$$R-Cl \xrightarrow{(aq.KOH)} R-OH\,(S_N \text{ reaction})$$

Statement (II) :



57. Given below are two statements :

Statement (I) : Molal depression constant K_f is given by $\frac{M_1RT_f}{\Delta S_{fus}}$, where symbols have their usual meaning.

Statement (II) : K_f for benzene is less than the K_f for water.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **Statement I** is incorrect but **Statement II** is correct

(2) Both **Statement I** and **Statement II** are incorrect

(3) Both **Statement I** and **Statement II** are correct

(4) **Statement I** is correct but **Statement II** is incorrect

Ans. (4)

Sol. Statement-I

Molar depression constant $k_f = \frac{M_1RT_f^2}{\Delta H_{fus}}$

$$k_f = \frac{M_1RT_f}{\left[\frac{\Delta H_{fus}}{T_f}\right]}$$

$$k_f = \frac{M_1RT_f}{\Delta S_{fus}}$$

Hence statement-I is correct

but k_f for benzene $= 5.12\frac{°C}{molal}$

k_f for water $= 1.86\frac{°C}{molal}$ Hence statement- II is incorrect

58. The IUPAC name of the following compound is –

```
             OH
             |
HC≡C–CH2–CH–CH2–CH=CH2
```

(1) 4-Hydroxyhept-1-en-6-yne

(2) 4- Hydroxyhept-6-en-1-yne

(3) Hept-6-en-1-yn-4-ol

(4) Hept-1-en-6-yn-4-ol

Ans. (4)

Sol.

```
             OH
             |
HC≡C–CH2–CH–CH2–CH=CH2
 7  6  5    4   3    2   1
```

Hept-1-en-6-yn-4-ol

59. Match **List-I** with **List-II** -

	List-I (Separation of)		List-II (Separation Technique)
(A)	Aniline from aniline-water mixture	(I)	Simple distillation
(B)	Glycerol from spent-lye in soap industry	(II)	Fractional distillation
(C)	Different fractions of crude oil in petroleum industry	(III)	Distillation at reduced pressure
(D)	Chloroform-Aniline mixture	(IV)	Steam distillation

Choose the **correct** answer from the options given below :

(1) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(2) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(4) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Ans. (1)

Sol. (A) Aniline – H_2O : Steam Distillation

(B) Glycerol from spent-lye in soap industry – Distillation under reduced pressure

(C) Different fraction of crude oil in petroleum industry – Fractional distillation

(D) $CHCl_3$ – Aniline – Simple distillation

60. A toxic compound "A" when reacted with NaCN in aqueous acidic medium yields an edible cooking component and food preservative 'B'. "B" is converted to "C" by diborane and can be used as an additive to petrol to reduce emission. "C" upon reaction with oleum at 140°C yields an inhalable anesthetic "D". Identify "A", "B", "C" and "D", respectively.

(1) Methanol; formaldehyde; methyl chloride; chloroform

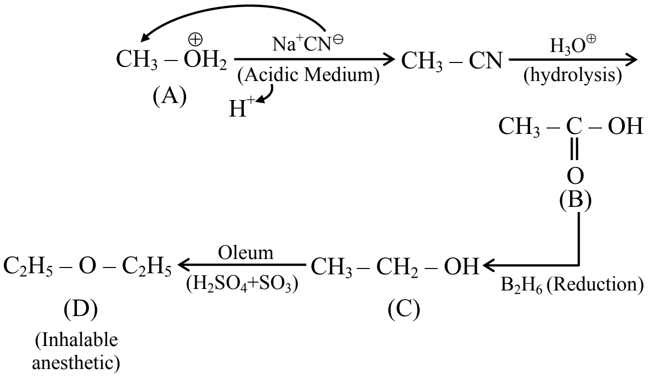
(2) Ethanol; acetonitrile; ethylamine; ethylene

(3) Methanol; acetic acid; ethanol; diethyl ether

(4) Acetaldehyde;2- hydroxypropanoic acid; propanoic acid; dipropyl ether

Ans. (3)

Sol. Methanol ⟶ Acetic Acid ⟶ Ethanol ⟶ Diethylether
(A) (B) (C) (D)



61. The correct order of $[FeF_6]^{3-}$, $[CoF_6]^{3-}$, $[Ni(CO)_4]$ and $[Ni(CN)_4]^{2-}$ complex species based on the number of unpaired electrons present is :

(1) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CN)_4]^{2-} > [Ni(CO)_4]$

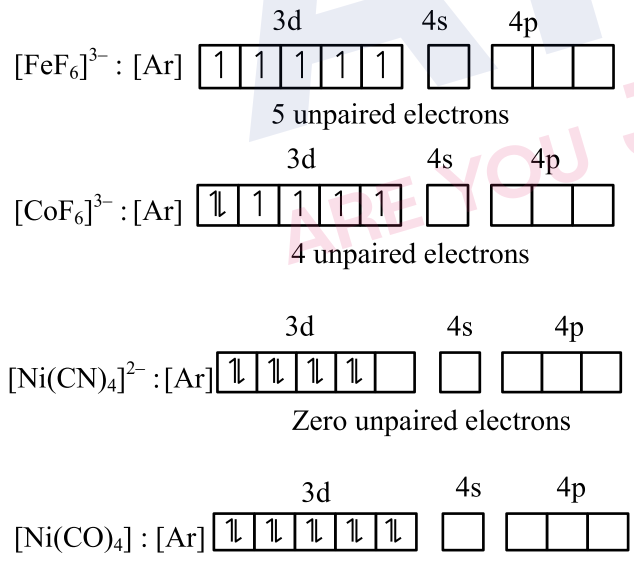
(2) $[Ni(CN)_4]^{2-} > [FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CO)_4]$

(3) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Ni(CO)_4] > [Ni(CN)_4]^{2-}$

(4) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CN)_4]^{2-} = [Ni(CO)_4]$

Ans. **(4)**

Sol.



Zero unpaired electrons
(electrons from 4s de-excited to 3d)

62. Consider the given data :

(a) $HCl(g) + 10H_2O(l) \rightarrow HCl.10\ H_2O$
$\Delta H = -69.01\ kJ\ mol^{-1}$

(b) $HCl(g) + 40H_2O(l) \rightarrow HCl.40\ H_2O$
$\Delta H = -72.79\ kJ\ mol^{-1}$

Choose the **correct** statement :

(1) Dissolution of gas in water is an endothermic process

(2) The heat of solution depends on the amount of solvent.

(3) The heat of dilution for the HCl ($HCl.10H_2O$ to $HCl.40H_2O$) is 3.78 kJ mol^{-1}.

(4) The heat of formation of HCl solution is represented by both (a) and (b)

Ans. **(2)**

Sol. From the given information

ΔH is negative so it means dissolution of gas HCl(g) is exothermic.

$HCl(g) + 10H_2O(l) \rightarrow HCl\ .\ 10H_2O$ .....(1)

$\Delta H_1 = -69.01 \dfrac{kJ}{mol}$

$HCl(g) + 40H_2O(l) \rightarrow HCl\ .\ 40H_2O$ .....(2)

$\Delta H_2 = -72.79 \dfrac{kJ}{mol}$

Hence heat of solution depends upon amount of solvent

By equation.....(2) – equation ....(1)

$HCl.10H_2O + 30H_2O(\ell) \rightarrow HCl.40H_2O$

So Heat of dilution = –72.79 – (–69.01)

$= -3.78 \dfrac{kJ}{mol}$

Hence option (3) is incorrect.

For heat of formation reactant should be in elemental form hence option (4) is incorrect

63. Consider the ground state of chromium atom (Z = 24). How many electrons are with Azimuthal quantum number $l = 1$ and $l = 2$ respectively ?

(1) 12 and 4 (2) 16 and 4

(3) 12 and 5 (4) 16 and 5

Ans. **(3)**

Sol. Cr : $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^5\ 4s^1$

$\ell=1$ (2p), $\ell=1$ (3p), $\ell=2$ (3d)

electrons having $\ell = 1 \Rightarrow 12$

electrons having $\ell = 2 \Rightarrow 5$

64. Given below are two statements :

Statement (I) : The first ionisation enthalpy of group 14 elements is higher than the corresponding elements of group 13.

Statement (II) : Melting points and boiling points of group 13 elements are in general much higher than those the corresponding elements of group 14.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **Statement I** is correct but **Statement II** is incorrect

(2) **Statement I** is incorrect but **Statement II** is correct

(3) Both **Statement I** and **Statement II** are incorrect

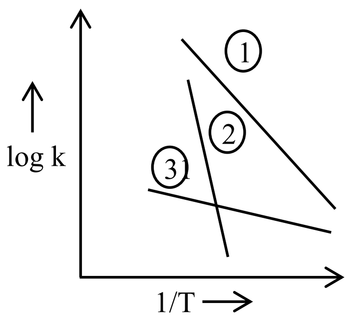
(4) Both **Statement I** and **Statement II** are correct

Ans. **(1)**

Sol. Statement 1 is correct since left to right 1E increases in general in periodic table.

Statement 2 is incorrect since M.P. of group 14 elements is more than group 13 elements.

65. Consider the following plots of log of rate constant k (log k) vs $\frac{1}{T}$ for three different reactions. The correct order of activation energies of these reactions is



(1) $Ea_2 > Ea_1 > Ea_3$ (2) $Ea_1 > Ea_3 > Ea_2$

(3) $Ea_1 > Ea_2 > Ea_3$ (4) $Ea_3 > Ea_2 > Ea_1$

Ans. **(1)**

Sol. $K = A\,e^{-Ea/RT}$

$$\log k = \log A - \frac{Ea}{2.303RT}$$

For graph between logk with $\frac{1}{T}$

$$|\text{Slope of curve}| = \frac{Ea}{2.303R}$$

From given graph

Magnitude of slope $\Rightarrow (2) > (1) > (3)$

Hence $Ea_2 > Ea_1 > Ea_3$

66. 'X' is the number of electrons in t_{2g} orbitals of the most stable complex ion among $[Fe(NH_3)_6]^{3+}$, $[Fe(Cl_6)]^{3-}$, $[Fe(C_2O_4)_3]^{3-}$ and $[Fe(H_2O)_6]^{3+}$. The nature of oxide of vanadium of the type V_2O_X is:

(1) Acidic

(2) Neutral

(3) Basic

(4) Amphoteric

Ans. **(4)**

Sol.

Most stable is $[Fe(C_2O_4)_3]^{3-}$ due to Chelation effect.

$[Fe(C_2O_4)_3]^{3-}$: [Ar] 3d: | 1 | 1 | 1 | 1 | 1 | 4s: | | 4p: | | | |

5 unpaired electrons ; x = 5

V_2O_5 is amphoteric.

67. The elements of Group 13 with highest and lowest first ionisation enthalpies are respectively:

(1) B & Ga

(2) B & Tl

(3) Tl & B

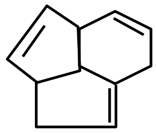
(4) B & In

Ans. **(4)**

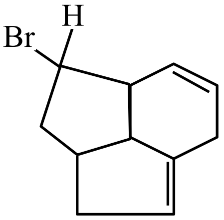
Sol. IE order

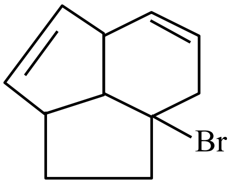
$B > T\ell > Ga > Al > In$

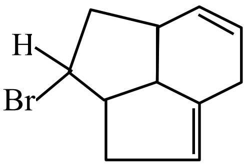
68. Consider the following molecule (X).

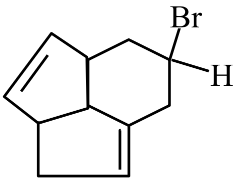
The structure of X is 

(X)

(1) 

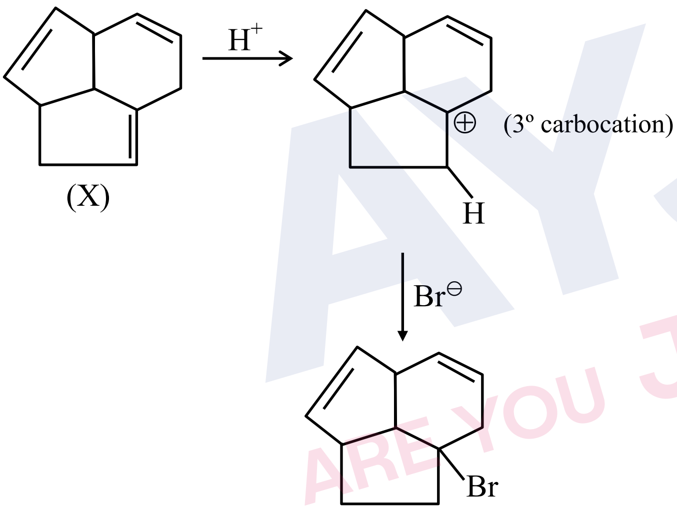
(2) 

(3) 

(4) 

Ans. (2)

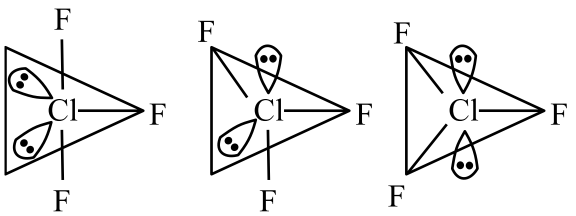
Sol.



(Major Product)

69. Given below are two statements:

Statement (I) : for $\ddot{C}\ell F_3$, all three possible structures may be drawn as follows.



I II III

Statement (II) : Structure III is most stable, as the orbitals having the lone pairs are axial, where the $\ell p - bp$ repulsion is minimum.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

(1) **Statement I** is incorrect but **statement II** is correct.

(2) **Statement I** is correct but **statement II** is incorrect.

(3) Both **Statement I** and **statement II** are correct.

(4) Both **Statement I** and **statement II** are incorrect.

Ans. (2)

Sol. Statement 1 is correct.

Statement 2 is incorrect since in sp^3d hybridization; lone pair cannot occupy axial position.

70. Half life of zero order reaction A → product is 1 hour, when initial concentration of reaction is 2.0 mol L^{-1}. The time required to decrease concentration of A from 0.50 to 0.25 mol L^{-1} is:

(1) 0.5 hour (2) 4 hour

(3) 15 min (4) 60 min

Ans. (3)

Sol. For zero order reaction

$$\text{Half life} = \frac{A_o}{2k}$$

$$60\,\text{min} = \frac{2}{2k}$$

$$k = \frac{1}{60}\ \text{M/min}$$

Now

$$A_t = A_o - kt$$

$$t = \frac{A_o - A_t}{k}$$

$$= \frac{0.5 - 0.25}{1/60}$$

$$0.25 \times 60$$

$$t = 15\ \text{min}$$

SECTION-B

71. Sea water, which can be considered as a 6 molar (6 M) solution of NaCl, has a density of 2 g mL^{-1}. The concentration of dissolved oxygen (O_2) in sea water is 5.8 ppm. Then the concentration of dissolved oxygen (O_2) in sea water, is x × 10^{-4} m.

x =_______. (Nearest integer)

Given: Molar mass of NaCl is 58.5 g mol^{-1}

Molar mass of O_2 is 32 g mol^{-1}

Ans. (2)

Sol. Sea water is 6 Molar in NaCl, So 1000 ml of sea water contains 6 mol of NaCl.

mass of solution = Volume × density

= 1000 × 2

mass of solution = 2000g

$$\text{ppm} = \frac{\text{mass of } O_2}{2000} \times 10^6$$

mass of O_2 = 5.8 × 2 × 10^{-3}

= 1.16 × 10^{-2} g

$$\text{molality for } O_2 = \frac{1.16\times10^{-2}/32}{(2000-6\times58.5)}\times1000$$

$$= \frac{1.16\times10}{32\times1649}$$

= 0.000219

= 2.19 × 10^{-4}

Correct answer ⇒ 2

72. The amount of calcium oxide produced on heating 150 kg limestone (75% pure) is ______kg. (Nearest integer)

Given : Molar mass (in g mol^{-1}) of Ca-40, O-16, C-12

Ans. (63)

Sol. $CaCO_3 \rightarrow CaO + CO_2$

$$\text{mass of } CaCO_3 = \frac{150\times75}{100} = 112.5\,\text{kg}$$

= 112500g

n_{CaCO_3} = 1125

So moles of CaO = 1125

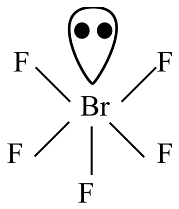
$$\text{mass of CaO} = \frac{1125\times56}{1000} = 63\,\text{kg}$$

Correct answer ⇒ 63

73. A metal complex with a formula $MC\ell_4.3NH_3$ is involved in sp^3d^2 hybridisation. It upon reaction with excess of $AgNO_3$ solution gives 'x' moles of AgCl. Consider 'x' is equal to the number of lone pairs of electron present in central atom of BrF_5. Then the number of geometrical isomers exhibited by the complex is________.

Ans. (2)

Sol.



1 lone pair hence 1 mole AgCl.

Complex is $[M(NH_3)_3Cl_3]Cl$.

It shows 2 geometrical isomers (Ma_3b_3 type) facial (fac) & meridional (Mer)

74. The molar conductance of an infinitely dilute solution of ammonium chloride was found to be 185 S cm^2 mol^{-1} and the ionic conductance of hydroxyl and chloride ions are 170 and 70 S cm^2 mol^{-1}, respectively. If molar conductance of 0.02 M solution of ammonium hydroxide is 85.5 S cm^2 mol^{-1}, its degree of dissociation is given by x × 10^{-1}. The value of x is_____. (Nearest integer)

Ans. (3)

Sol. λ°_m of NH_4Cl = 185

$$(\lambda^\circ_m)_{NH_4^+} + (\lambda^\circ_m)_{Cl^-} = 185$$

$$(\lambda^\circ_m)_{NH_4^+} = 185 - 70 = 115\ \text{Scm}^2\ \text{mol}^{-1}$$

$$(\lambda^\circ_m)_{NH_4OH} = (\lambda^\circ_m)_{NH_4^+} + (\lambda^\circ_m)_{OH^-}$$

= 115 + 170

$$(\lambda^\circ_m)_{NH_4OH} = 285\ \text{Scm}^2\text{mol}^{-1}$$

$$\text{degree of dissociation} = \frac{(\lambda_m)_{NH_4OH}}{(\lambda^\circ_m)_{NH_4OH}}$$

$$= \frac{85.5}{285}$$

= 0.3

= 3 × 10^{-1}

75. x mg of $Mg(OH)_2$ (molar mass = 58) is required to be dissolved in 1.0 L of water to produce a pH of 10.0 at 298 K. The value of x is____ mg. (Nearest integer)

(Given : $Mg(OH)_2$ is assumed to dissociate completely in H_2O)

Ans. **(3)**

Sol. pH = 10

pOH = 4

$[OH^-] = 10^{-4}$

no. of moles of $OH^- = 10^{-4}$

no. of moles of $Mg(OH)_2 = \frac{10^{-4}}{2} = 5\times10^{-5}$

mass of $Mg(OH)_2 = 5 \times 10^{-5} \times 58 \times 10^3$ mg

$= 2.9$