

FINAL JEE-MAIN EXAMINATION – APRIL, 2024

(Held On Monday 08th April, 2024)

TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

SECTION-A

1. The value of $k \in \mathbb{N}$ for which the integral

$$I_n = \int_0^1 (1-x^k)^n dx, \quad n \in \mathbb{N}, \text{ satisfies } 147 I_{20} = 148 I_{21}$$

is :

- (1) 10 (2) 8
(3) 14 (4) 7

Ans. (4)

Sol. $I_n = \int_0^1 (1-x^k)^n \cdot 1 \, dx$

$$I_n = (1-x^k)^n \cdot x - nk \int_0^1 (1-x^k)^{n-1} \cdot x^{k-1} \cdot dx$$

$$I_n = nk \int_0^1 [(1-x^k)^n - (1-x^k)^{n-1}] dx$$

$$I_n = nk I_n - nk I_{n-1}$$

$$\frac{I_n}{I_{n-1}} = \frac{nk}{nk+1}$$

$$\frac{I_{21}}{I_{20}} = \frac{21k}{1+21k}$$

$$= \frac{147}{148} \Rightarrow k = 7$$

2. The sum of all the solutions of the equation $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$ is :

- (1) $1 + \log_6(8)$ (2) $\log_8(6)$
(3) $1 + \log_8(6)$ (4) $\log_8(4)$

Ans. (3)

Sol. $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$

Put $8^x = t$

$$t^2 - 16 + 48 = 0$$

$$\Rightarrow t = 4 \text{ or } t = 12$$

$$\Rightarrow 8^x = 4 \quad 8^x = 12$$

$$\Rightarrow x = \log_8 4 \quad x = \log_8 12$$

$$\text{sum of solution} = \log_8 4 + \log_8 12$$

$$= \log_8 48 = \log_8(6 \cdot 8)$$

$$= 1 + \log_8 6$$

TEST PAPER WITH SOLUTION

3. Let the circles $C_1 : (x - \alpha)^2 + (y - \beta)^2 = r_1^2$ and

$$C_2 : (x - 8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2 \text{ touch each other}$$

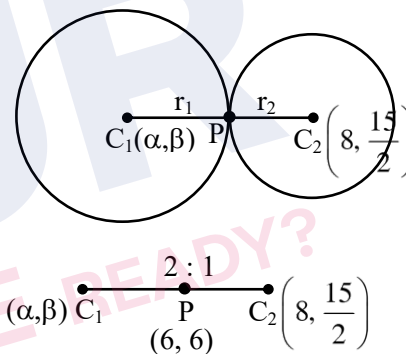
externally at the point $(6, 6)$. If the point $(6, 6)$ divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio $2 : 1$, then

$$(\alpha + \beta) + 4(r_1^2 + r_2^2) \text{ equals}$$

- (1) 110 (2) 130
(3) 125 (4) 145

Ans. (2)

Sol.



$$\therefore \frac{16 + \alpha}{3} = 6 \text{ and } \frac{15 + \beta}{3} = 6$$

$$\Rightarrow (\alpha, \beta) \equiv (2, 3)$$

$$\text{Also, } C_1 C_2 = r_1 + r_2$$

$$\Rightarrow \sqrt{(2-8)^2 + \left(3 - \frac{15}{2}\right)^2} = 2r_1 + r_2$$

$$\Rightarrow r_2 = \frac{5}{2} \Rightarrow r_1 = 2r_2 = 5$$

$$\therefore (\alpha + \beta) + 4(r_1^2 + r_2^2)$$

$$= 5 + 4\left(\frac{25}{4} + 25\right) = 130$$

4. Let $P(x, y, z)$ be a point in the first octant, whose projection in the xy -plane is the point Q . Let $OP = \gamma$; the angle between OQ and the positive x -axis be θ ; and the angle between OP and the positive z -axis be ϕ , where O is the origin. Then the distance of P from the x -axis is :

(1) $\gamma\sqrt{1 - \sin^2 \phi \cos^2 \theta}$ (2) $\gamma\sqrt{1 + \cos^2 \theta \sin^2 \phi}$
(3) $\gamma\sqrt{1 - \sin^2 \theta \cos^2 \phi}$ (4) $\gamma\sqrt{1 + \cos^2 \phi \sin^2 \theta}$

Ans. (1)

Sol. $P(x, y, z), Q(x, y, 0); x^2 + y^2 + z^2 = \gamma^2$

$$\overrightarrow{OQ} = x\hat{i} + y\hat{j}$$

$$\cos \theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\Rightarrow \sin^2 \phi = \frac{x^2 + y^2}{x^2 + y^2 + z^2}$$

$$\text{distance of } P \text{ from } x\text{-axis} = \sqrt{y^2 + z^2}$$

$$\Rightarrow \sqrt{\gamma^2 - x^2} \Rightarrow \gamma \sqrt{1 - \frac{x^2}{\gamma^2}}$$

$$= \gamma \sqrt{1 - \cos^2 \theta \sin^2 \phi}$$

5. The number of critical points of the function

$$f(x) = (x - 2)^{2/3} (2x + 1) \text{ is :}$$

- (1) 2 (2) 0
(3) 1 (4) 3

Ans. (1)

Sol. $f(x) = (x - 2)^{2/3} (2x + 1)$

$$f'(x) = \frac{2}{3}(x - 2)^{-1/3} (2x + 1) + (x - 2)^{2/3} (2)$$

$$f'(x) = 2 \times \frac{(2x + 1) + (x - 2)}{3(x - 2)^{1/3}}$$

$$\frac{3x - 1}{(x - 2)^{1/3}} = 0$$

$$\text{Critical points } x = \frac{1}{3} \text{ and } x = 2$$

6. Let $f(x)$ be a positive function such that the area bounded by $y = f(x), y = 0$ from $x = 0$ to $x = a > 0$ is $e^{-a} + 4a^2 + a - 1$. Then the differential equation, whose general solution is $y = c_1 f(x) + c_2$, where c_1 and c_2 are arbitrary constants, is :

(1) $(8e^x - 1) \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$

(2) $(8e^x + 1) \frac{d^2 y}{dx^2} - \frac{dy}{dx} = 0$

(3) $(8e^x + 1) \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$

(4) $(8e^x - 1) \frac{d^2 y}{dx^2} - \frac{dy}{dx} = 0$

Ans. (3)

Sol. $\int_0^a f(x) dx = e^{-a} + 4a^2 + a - 1$

$$f(a) = -e^{-a} + 8a + 1$$

$$f(x) = -e^{-x} + 8x + 1$$

$$\text{Now } y = C_1 f(x) + C_2$$

$$\frac{dy}{dx} = C_1 f'(x) = C_1 (e^{-x} + 8) \quad \dots (1)$$

$$\frac{d^2 y}{dx^2} = -C_1 e^{-x} \Rightarrow -e^x \frac{d^2 y}{dx^2}$$

Put in equation (1)

$$\frac{dy}{dx} = -e^x \frac{d^2 y}{dx^2} (e^{-x} + 8)$$

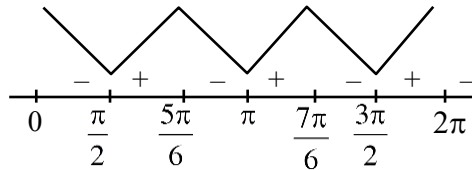
$$(8e^x + 1) \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$$

7. Let $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$. The number of points of local maxima of f in interval $(0, 2\pi)$ is:

- (1) 1 (2) 2
(3) 3 (4) 4

Ans. (2)

Sol. $f(x) = 4\cos^3(x) + 3\sqrt{3}\cos^2(x) - 10$; $x \in (0, 2\pi)$
 $\Rightarrow f'(x) = 12\cos^2 x [-\sin(x)] + 3\sqrt{3}(2\cos(x))[-\sin(x)]$
 $\Rightarrow f'(x) = -6\sin(x)\cos(x)[2\cos(x) + \sqrt{3}]$



local maxima at $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

8. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where

I is the identity matrix of order 3×3 , then $2a + 3b$ is equal to :

- (1) -10 (2) -13
(3) -9 (4) -12

Ans. (2)

Sol. $A^3 - 4A^2 + A + 21I = 0$
 $\text{tr}(A) = 4 = 5 + 6 \Rightarrow b = -1$
 $|A| = -21$
 $-16 + a = -21 \Rightarrow a = -5$
 $2a + 3b = -13$

9. If the shortest distance between the lines

$$L_1 : \vec{r} = (2 + \lambda)\hat{i} + (1 - 3\lambda)\hat{j} + (3 + 4\lambda)\hat{k}, \lambda \in \mathbb{R}$$

$$L_2 : \vec{r} = 2(1 + \mu)\hat{i} + 3(1 + \mu)\hat{j} + (5 + \mu)\hat{k}, \mu \in \mathbb{R}$$

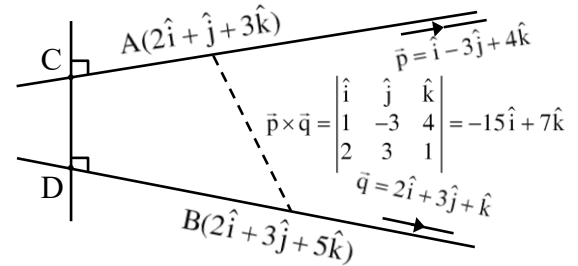
is $\frac{m}{\sqrt{n}}$, where $\gcd(m, n) = 1$, then the value of

$m + n$ equals.

- (1) 384 (2) 387
(3) 377 (4) 390

Ans. (2)

Sol.



$$\begin{aligned} \text{Shortest distance (CD)} &= \frac{|\overline{AB} \cdot \vec{p} \times \vec{q}|}{|\vec{p} \times \vec{q}|} \\ &= \frac{|(0\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (-15\hat{i} + 7\hat{j} + 7\hat{k})|}{\sqrt{355}} \\ &= \frac{0 + 14 + 18}{\sqrt{355}} = \frac{32}{\sqrt{355}} \end{aligned}$$

$$\therefore m + n = 32 + 355 = 387$$

10. Let the sum of two positive integers be 24. If the probability, that their product is not less than $\frac{3}{4}$ times their greatest positive product, is $\frac{m}{n}$,

where $\gcd(m, n) = 1$, then $n - m$ equals :

- (1) 9 (2) 11
(3) 8 (4) 10

Ans. (4)

Sol. $x + y = 24, x, y \in \mathbb{N}$

$$AM > GM \Rightarrow xy \leq 144$$

$$xy \geq 108$$

Favorable pairs of (x, y) are

$(13, 11), (12, 12), (14, 10), (15, 9), (16, 8),$
 $(17, 7), (18, 6), (6, 18), (7, 17), (8, 16), (9, 15),$
 $(10, 14), (11, 13)$

i.e. 13 cases

Total choices for $x + y = 24$ is 23

$$\text{Probability} = \frac{13}{23} = \frac{m}{n}$$

$$n - m = 10$$

11. If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$,

then $80(\tan^2 x - \cos x)$ is equal to :

- (1) 109 (2) 108
(3) 18 (4) 19

Ans. (1)

Sol. $\sin x = -\frac{3}{5}$, $\pi < x < \frac{3\pi}{2}$

$$\tan x = \frac{3}{4} \quad \cos x = -\frac{4}{5}$$

$$80(\tan^2 x - \cos x)$$

$$= 80\left(\frac{9}{16} + \frac{4}{5}\right) = 45 + 64 = 109$$

12. Let $I(x) = \int \frac{6}{\sin^2 x(1 - \cot x)^2} dx$. If $I(0) = 3$, then

$I\left(\frac{\pi}{12}\right)$ is equal to :

- (1) $\sqrt{3}$ (2) $3\sqrt{3}$
(3) $6\sqrt{3}$ (4) $2\sqrt{3}$

Ans. (2)

Sol. $I(x) = \int \frac{6dx}{\sin^2 x(1 - \cot x)^2} = \int \frac{6 \operatorname{cosec}^2 x dx}{(1 - \cot x)^2}$

Put $1 - \cot x = t$

$\operatorname{cosec}^2 x dx = dt$

$$I = \int \frac{6dt}{t^2} = \frac{-6}{t} + c$$

$$I(x) = \frac{-6}{1 - \cot x} + c, c = 3$$

$$I(x) = 3 - \frac{6}{1 - \cot x}, I\left(\frac{\pi}{12}\right) = 3 - \frac{6}{1 - (2 + \sqrt{3})}$$

$$I\left(\frac{\pi}{12}\right) = 3 + \frac{6}{\sqrt{3} + 1} = 3 + \frac{6(\sqrt{3} - 1)}{2} = 3\sqrt{3} + 2$$

13. The equations of two sides AB and AC of a triangle ABC are $4x + y = 14$ and $3x - 2y = 5$,

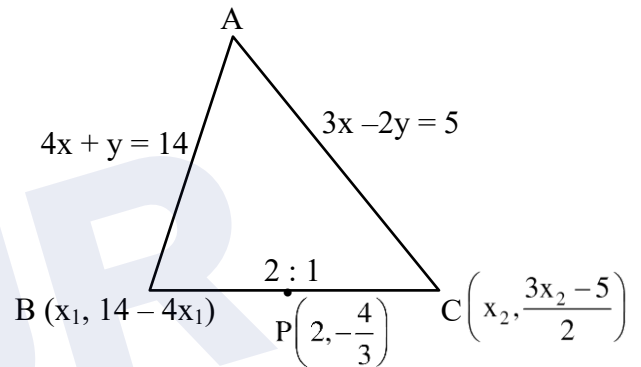
respectively. The point $\left(2, -\frac{4}{3}\right)$ divides the third

side BC internally in the ratio 2 : 1. The equation of the side BC is :

- (1) $x - 6y - 10 = 0$ (2) $x - 3y - 6 = 0$
(3) $x + 3y + 2 = 0$ (4) $x + 6y + 6 = 0$

Ans. (3)

Sol.



$$\frac{2x_2 + x_1}{3} = 2, \frac{2\left(\frac{3x_2 - 5}{2}\right) + (14 - 4x_1)}{3} = \frac{-4}{3}$$

$$2x_2 + x_1 = 6, 3x_2 - 4x_1 = -13$$

$$x_2 = 1, x_1 = 4$$

So, $C(1, -1), B(4, -2)$

$$m = \frac{-1}{3}$$

Equation of BC : $y + 1 = \frac{-1}{3}(x - 1)$

$$3y + 3 = -x + 1$$

$$x + 3y + 2 = 0$$

14. Let $[t]$ be the greatest integer less than or equal to t . Let A be the set of all prime factors of 2310 and

$$f: A \rightarrow \mathbb{Z} \text{ be the function } f(x) = \left\lceil \log_2 \left(x^2 + \left\lceil \frac{x^3}{5} \right\rceil \right) \right\rceil.$$

The number of one-to-one functions from A to the range of f is :

- (1) 20 (2) 120
(3) 25 (4) 24

Ans. (2)

Sol. $N = 2310 = 231 \times 10$

$$= 3 \times 11 \times 7 \times 2 \times 5$$

$$A = \{2, 3, 5, 7, 11\}$$

$$f(x) = \left\lceil \log_2 \left(x^2 + \left\lceil \frac{x^3}{5} \right\rceil \right) \right\rceil$$

$$f(2) = [\log_2(5)] = 2$$

$$f(3) = [\log_2(14)] = 3$$

$$f(5) = [\log_2(25 + 25)] = 5$$

$$f(7) = [\log_2(117)] = 6$$

$$f(11) = [\log_2 387] = 8$$

$$\text{Range of } f: B = \{2, 3, 5, 6, 8\}$$

$$\text{No. of one-one functions} = 5! = 120$$

15. Let z be a complex number such that $|z + 2| = 1$

and $\text{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value of $|\text{Re}(\overline{z+2})|$

is :

- (1) $\frac{\sqrt{6}}{5}$ (2) $\frac{1+\sqrt{6}}{5}$
(3) $\frac{24}{5}$ (4) $\frac{2\sqrt{6}}{5}$

Ans. (4)

Sol. $|z + 2| = 1, \text{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$

$$\text{Let } z + 2 = \cos\theta + i\sin\theta$$

$$\frac{1}{z+2} = \cos\theta - i\sin\theta$$

$$\Rightarrow \frac{z+1}{z+2} = 1 - \frac{1}{z+2} = 1 - (\cos\theta - i\sin\theta)$$

$$= (1 - \cos\theta) + i\sin\theta$$

$$\text{Im}\left(\frac{z+1}{z+2}\right) = \sin\theta, \sin\theta = \frac{1}{5}$$

$$\cos\theta = \pm \sqrt{1 - \frac{1}{25}} = \pm \frac{2\sqrt{6}}{5}$$

$$|\text{Re}(\overline{z+2})| = \frac{2\sqrt{6}}{5}$$

16. If the set $R = \{(a, b) ; a + 5b = 42, a, b \in \mathbb{N}\}$

has m elements and $\sum_{n=1}^m (1 + i^{n!}) = x + iy$, where

$I = \sqrt{-1}$, then the value of $m + x + y$ is :

- (1) 8 (2) 12
(3) 4 (4) 5

Ans. (2)

Sol. $a + 5b = 42, a, b \in \mathbb{N}$

$$a = 42 - 5b, b = 1, a = 37$$

$$b = 2, a = 32$$

$$b = 3, a = 27$$

\vdots

$$b = 8, a = 2$$

$$R \text{ has "8" elements} \Rightarrow m = 8$$

$$\sum_{n=1}^8 (1 - i^{n!}) = x + iy$$

$$\text{for } n \geq 4, i^{n!} = 1$$

$$\Rightarrow (1 - i) + (1 - i^{2!}) + (1 - i^{3!})$$

$$= 1 - I + 2 + 1 + 1$$

$$= 5 - I = x + iy$$

$$m + x + y = 8 + 5 - 1 = 12$$

17. For the function $f(x) = (\cos x) - x + 1$, $x \in \mathbb{R}$, between the following two statements
(S1) $f(x) = 0$ for only one value of x is $[0, \pi]$.

(S2) $f(x)$ is decreasing in $\left[0, \frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2}, \pi\right]$.

- (1) Both (S1) and (S2) are correct
(2) Only (S1) is correct
(3) Both (S1) and (S2) are incorrect
(4) Only (S2) is correct

Ans. (2)

Sol. $f(x) = \cos x - x + 1$

$$f'(x) = -\sin x - 1$$

f is decreasing $\forall x \in \mathbb{R}$

$$f(x) = 0$$

$$f(0) = 2, f(\pi) = -\pi$$

f is strictly decreasing in $[0, \pi]$ and $f(0) \cdot f(\pi) < 0$

\Rightarrow only one solution of $f(x) = 0$

S1 is correct and S2 is incorrect.

18. The set of all α , for which the vector $\vec{a} = \alpha\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$ are inclined at an obtuse angle for all $t \in \mathbb{R}$ is :

- (1) $[0, 1)$ (2) $(-2, 0]$
(3) $\left(-\frac{4}{3}, 0\right]$ (4) $\left(-\frac{4}{3}, 1\right)$

Ans. (3)

Sol. $\vec{a} = \alpha\hat{i} + 6\hat{j} - 3\hat{k}$

$$\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$$

so $\vec{a} \cdot \vec{b} < 0$, $\forall t \in \mathbb{R}$

$$\alpha t^2 - 12 + 6\alpha t < 0$$

$$\alpha t^2 + 6\alpha t - 12 < 0, \forall t \in \mathbb{R}$$

$$\alpha < 0, \text{ and } D < 0$$

$$36\alpha^2 + 48\alpha < 0$$

$$12\alpha(3\alpha + 4) < 0$$

$$\frac{-4}{3} < \alpha < 0$$

also for $\alpha = 0$, $\vec{a} \cdot \vec{b} < 0$

$$\text{hence } \alpha \in \left(-\frac{4}{3}, 0\right]$$

19. Let $y = y(x)$ be the solution of the differential equation $(1 + y^2)e^{\tan x} dx + \cos^2 x (1 + e^{2\tan x}) dy = 0$, $y(0) = 1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to :

- (1) $\frac{2}{e}$ (2) $\frac{1}{e^2}$
(3) $\frac{1}{e}$ (4) $\frac{2}{e^2}$

Ans. (3)

Sol. $(1 + y^2)e^{\tan x} dx + \cos^2 x (1 + e^{2\tan x}) dy = 0$

$$\int \frac{\sec^2 x e^{\tan x}}{1 + e^{2\tan x}} dx + \int \frac{dy}{1 + y^2} = C$$

$$\Rightarrow \tan^{-1}(e^{\tan x}) + \tan^{-1} y = C$$

$$\text{for } x = 0, y = 1, \tan^{-1}(1) + \tan^{-1} 1 = C$$

$$C = \frac{\pi}{2}$$

$$\tan^{-1}(e^{\tan x}) + \tan^{-1} y = \frac{\pi}{2}$$

$$\text{Put } x = \pi, \tan^{-1} e + \tan^{-1} y = \frac{\pi}{2}$$

$$\tan^{-1} y = \cot^{-1} e$$

$$y = \frac{1}{e}$$

20. Let $H : \frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6)$, $\alpha > 0$ lies on H . If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2 + \beta$ is equal to :

- (1) 170 (2) 171
(3) 169 (4) 172

Ans. (2)

Sol. $H: \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1, e = \sqrt{3}$

$$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{3} \Rightarrow \frac{a^2}{b^2} = 2$$

$$a^2 = 2b^2$$

$$\text{length of L.R.} = \frac{2a^2}{b} = 4\sqrt{3}$$

$$a = \sqrt{6}$$

$$P(\alpha, 6) \text{ lie on } \frac{y^2}{3} - \frac{x^2}{6} = 1$$

$$12 - \frac{\alpha^2}{6} = 1 \Rightarrow \alpha^2 = 66$$

$$\text{Foci} = (0, \pm be) = (0, 3) \text{ \& } (0, -3)$$

Let d_1 & d_2 be focal distances of $P(\alpha, 6)$

$$d_1 = \sqrt{\alpha^2 + (6 + be)^2}, d_2 = \sqrt{\alpha^2 + (6 - be)^2}$$

$$d_1 = \sqrt{66 + 81}, d_2 = \sqrt{66 + 9}$$

$$\beta = d_1 d_2 = \sqrt{147 \times 75} = 105$$

$$\alpha^2 + \beta = 66 + 105 = 171$$

SECTION-B

- 21.** Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n , then n is equal to _____.

Ans. (7)

Sol. $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$$

$$A^7 = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix} \begin{bmatrix} -54 & 27 \\ -27 & -27 \end{bmatrix} = \begin{bmatrix} 3^6 \times 2 & -27^2 \\ 27^2 & 3^6 \end{bmatrix}$$

$$3^7 = 3^n \Rightarrow n = 7$$

- 22.** If the orthocentre of the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$, is the centroid of another triangle, whose circumcentre and orthocentre respectively are $(3, 4)$ and $(-6, -8)$, then the value of $|a - b|$ is _____.

Ans. (16)

Sol. $2x + 3y - 1 = 0$

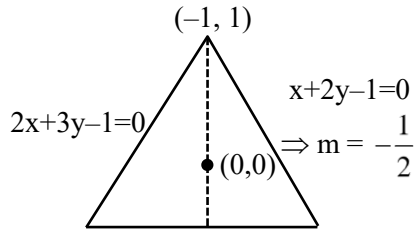
$$x + 2y - 1 = 0$$

$$ax + by - 1 = 0$$

$$\begin{array}{c} \text{2} \qquad \qquad \text{1} \\ \text{---} \text{---} \text{---} \\ (-6, -8) \quad \text{G} \quad (3, 4) \\ \text{H} \qquad \qquad (6, 6) \end{array}$$

$$\left(\frac{6-6}{3}, \frac{8-8}{3} \right)$$

$$= (0, 0)$$



$$ax + by - 1 = 0$$

$$\left(\frac{1-0}{-1-0} \right) \left(\frac{-a}{b} \right) = -1$$

$$\Rightarrow -a = b$$

$$\Rightarrow ax - ay - 1 = 0$$

$$ax - a \left(1 - \frac{2x}{3} \right) - 1 = 0$$

$$x \left(a + \frac{2a}{3} \right) = \frac{a}{3}$$

$$x = \frac{a+3}{5a}$$

$$2 \left(\frac{a+3}{5a} \right) + 3y - 1 = 0$$

$$y = \frac{1 - \frac{2a+6}{5a}}{3} = \frac{3a-6}{3 \times 5a}$$

$$y = \frac{a-2}{5a}$$

$$\frac{\left(\frac{a-2}{5a} \right)}{\left(\frac{a+3}{5a} \right)} = 2 \Rightarrow a-2 = 2a+6$$

$$a = -8$$

$$b = 8$$

$$-8x + 8y - 1 = 0$$

$$|a - b| = 16$$

23. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and Yellow balls. If \bar{X} and \bar{Y} are the means of X and Y respectively, then $7\bar{X} + 4\bar{Y}$ is equal to _____.

Ans. (17)

Sol.

Blue balls	0	1	2	3	4	5
Prob.	$\frac{{}^5C_0 \cdot {}^4C_1}{{}^9C_3}$	$\frac{{}^5C_1 \cdot {}^4C_2}{{}^9C_3}$	$\frac{{}^5C_2 \cdot {}^4C_1}{{}^9C_3}$	$\frac{{}^5C_3 \cdot {}^4C_0}{{}^9C_3}$	0	0

$$7\bar{X} = \frac{{}^5C_1 \cdot {}^4C_2 + {}^5C_2 \cdot {}^4C_1 \times 2 + {}^5C_3 \cdot {}^4C_0 \times 3}{{}^9C_3} \times 7$$

$$\frac{30 + 80 + 30}{84} \times 7 = \frac{140}{12} = \frac{70}{6} = \frac{35}{3}$$

yellow	0	1	2	3	4
		${}^5C_2 \cdot {}^4C_1$	${}^5C_1 \cdot {}^4C_2$	${}^5C_0 \cdot {}^4C_3$	0

$$4\bar{Y} = \frac{40 + 60 + 12}{84} \times 4 = \frac{112}{21} = \frac{16}{3}$$

24. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to _____.

Ans. (36)

Sol. 2, 3, 4, 5, 7

total number of three digit numbers not divisible by 3 will be formed by using the digits

(4, 5, 7)

(3, 4, 7)

(2, 5, 7)

(2, 4, 7)

(2, 4, 5)

(2, 3, 5)

number of ways = $6 \times 3! = 36$

25. Let the positive integers be written in the form :

```

      1
    2   3
  4   5   6
. 7   8   9   10 .
. . . . .

```

If the k^{th} row contains exactly k numbers for every natural number k , then the row in which the number 5310 will be, is _____.

Ans. (103)

Sol. $S = 1 + 2 + 4 + 7 + \dots + T_n$

$$S = 1 + 2 + 4 + \dots$$

$$T_n = 1 + 1 + 2 + 3 + \dots + (T_n - T_{n-1})$$

$$T_n = 1 + \left(\frac{n-1}{2} \right) [2 + (n-2) \times 1]$$

$$T_n = 1 + 1 + \frac{n(n-1)}{2}$$

$$n = 100 \quad T_n = 1 + \frac{100 \times 99}{2} = 4950 + 1$$

$$n = 101 \quad T_n = 1 + \frac{101 \times 100}{2} = 5050 + 1 = 5051$$

$$n = 102 \quad T_n = 1 + \frac{102 \times 101}{2} = 5151 + 1 = 5152$$

$$n = 103 \quad T_n = 1 + \frac{103 \times 102}{2} = 5254$$

$$n = 104 \quad T_n = 1 + \frac{104 \times 103}{2} = 5357$$

26. If the range of $f(\theta) = \frac{\sin^4 \theta + 3 \cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$, $\theta \in \mathbb{R}$ is

$[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to

Ans. (96)

Sol. $f(\theta) = \frac{\sin^4 \theta + 3 \cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$

$$f(\theta) = 1 + \frac{2 \cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$$

$$f(\theta) = \frac{2 \cos^2 \theta}{\cos^4 \theta - \cos^2 \theta + 1} + 1$$

$$f(\theta) = \frac{2}{\cos^2 \theta + \sec^2 \theta - 1} + 1$$

$$f(\theta)|_{\min.} = 1$$

$$f(\theta)|_{\max.} = 3$$

$$S = \frac{64}{1 - 1/3} = 96$$

27. Let $\alpha = \sum_{r=0}^n (4r^2 + 2r + 1)^n C_r$

$$\text{and } \beta = \left(\sum_{r=0}^n \frac{{}^n C_r}{r+1} \right) + \frac{1}{n+1}. \text{ If } 140 < \frac{2\alpha}{\beta} < 281,$$

then the value of n is _____.

Ans. (5)

Sol. $\alpha = \sum_{r=0}^n (4r^2 + 2r + 1)^n C_r$

$$\alpha = 4 \sum_{r=0}^n r^2 \cdot \frac{n}{r} \cdot {}^{n-1} C_{r-1} + 2 \sum_{r=0}^n r \cdot \frac{n}{r} \cdot {}^{n-1} C_{r-1} + \sum_{r=0}^n {}^n C_r$$

$$+ 4n \sum_{r=0}^n {}^{n-1} C_{r-1} + 2n \sum_{r=0}^n {}^{n-1} C_{r-1} + \sum_{r=0}^n {}^n C_r$$

$$\alpha = 4n(n-1) \cdot 2^{n-2} + 4n \cdot 2^{n-1} + 2n \cdot 2^{n-1} + 2^n$$

$$\alpha = 2^{n-2} [4n(n-1) + 8n + 4n + 4]$$

$$\alpha = 2^{n-2} [4n^2 + 8n + 4]$$

$$\alpha = 2n(n+1)^2$$

$$\beta = \sum_{r=0}^n \frac{{}^n C_r}{r+1} + \frac{1}{n+1}$$

$$= \sum_{r=0}^n \frac{{}^{n+1} C_{r+1}}{n+1} + \frac{1}{n+1}$$

$$= \frac{1}{n+1} (1 + {}^{n+1} C_1 + \dots + {}^{n+1} C_{n+1})$$

$$= \frac{2^{n+1}}{n+1}$$

$$\frac{2\alpha}{\beta} = \frac{2^{n+1}(n+1)^2}{2^{n+1}} \cdot (n+1) = (n+1)^3$$

$$140 < (n+1)^3 < 281$$

$$n = 4 \Rightarrow (n+1)^3 = 125$$

$$n = 5 \Rightarrow (n+1)^3 = 216$$

$$n = 6 \Rightarrow (n+1)^3 = 343$$

$$\therefore n = 5$$

28. Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$, $\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$ and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectors. If \vec{r} is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ and $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$, then $\frac{|593\vec{r} + 67\vec{a}|^2}{(593)^2}$ is equal to _____.

Ans. (569)

Sol. $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$

$\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$

$\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$

$\vec{b} + \vec{c} = 20\hat{i} + 5\hat{j} - 12\hat{k}$

$\vec{b} - \vec{c} = -14\hat{i} + 9\hat{j} - 14\hat{k}$

$(\vec{r} - (\vec{b} + \vec{c})) \times \vec{a} = 0$

$\vec{r} - (\vec{b} + \vec{c}) = \lambda \vec{a}$

$\vec{r} = \lambda \vec{a} + \vec{b} + \vec{c}$

But $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow (\lambda \vec{a} + \vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow \lambda \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b} - \lambda \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c} - \vec{c} \cdot \vec{c} = 0$

$\lambda = \frac{\vec{c} \cdot \vec{c} - \vec{b} \cdot \vec{b}}{\vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}} = \frac{294 - 227}{-389 - 204} = \frac{-67}{593}$

$\therefore \vec{r} = \vec{b} + \vec{c} - \frac{67}{593} \vec{a}$

$\Rightarrow 593\vec{r} + 67\vec{a} = 593(\vec{b} + \vec{c})$

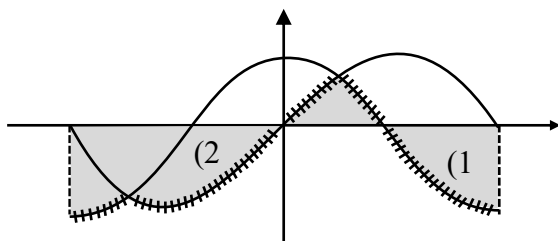
$\Rightarrow |\vec{b} + \vec{c}|^2 = 569$

29. Let the area of the region enclosed by the curve $y = \min\{\sin x, \cos x\}$ and the x-axis between $x = -\pi$ to $x = \pi$ be A. Then A^2 is equal to _____.

Ans. (16)

Sol. $y = \min\{\sin x, \cos x\}$

x-axis x = -\pi x = \pi



$$\int_0^{\pi/4} \sin x = (\cos x)_{\pi/4}^0 = 1 - \frac{1}{\sqrt{2}}$$

$$\int_{-\pi}^{-3\pi/4} (\sin x - \cos x) = (-\cos x - \sin x)_{-\pi}^{-3\pi/4}$$

$$= (\cos x + \sin x)_{-3\pi/4}^{-\pi}$$

$$= (-1 + 0) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= -1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$\int_{\pi/4}^{\pi/2} \cos x dx = (\sin x)_{\pi/4}^{\pi/2} = 1 - \frac{1}{\sqrt{2}}$$

A = 4

A² = 16

30. The value of

$$\lim_{x \rightarrow 0} 2 \left(\frac{1 - \cos x \sqrt{\cos 2x} \sqrt[3]{\cos 3x} \dots \sqrt[10]{\cos 10x}}{x^2} \right) \text{ is}$$

_____.

Ans. (55)

Sol.

$$\lim_{x \rightarrow 0} 2 \left(\frac{\left(1 - \left(1 - \frac{x^2}{2!}\right)\right) \left(1 - \frac{4x^2}{2!}\right) \left(1 - \frac{9x^2}{2!}\right) \dots \left(1 - \frac{100x^2}{2!}\right)}{x^2} \right)$$

By expansion

$$\lim_{x \rightarrow 0} \frac{2 \left(1 - \left(1 - \frac{x^2}{2}\right)\right) \left(1 - \frac{1}{2} \cdot \frac{4x^2}{2}\right) \left(1 - \frac{1}{3} \cdot \frac{9x^2}{2}\right) \dots \left(1 - \frac{1}{10} \cdot \frac{100x^2}{2}\right)}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{2 \left(1 - \left(1 - \frac{x^2}{2}\right)\right) \left(1 - \frac{2x^2}{2}\right) \left(1 - \frac{3x^2}{2}\right) \dots \left(1 - \frac{10x^2}{2}\right)}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{2 \left(1 - 1 + x^2 \left(\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \dots + \frac{10}{2}\right)\right)}{x^2}$$

$$2 \left(\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \dots + \frac{10}{2}\right)$$

$$1 + 2 + \dots + 10 = \frac{10 \times 11}{2} = 55$$

PHYSICS

SECTION-A

31. Three bodies A, B and C have equal kinetic energies and their masses are 400 g, 1.2 kg and 1.6 kg respectively. The ratio of their linear momenta is :

- (1) $1:\sqrt{3}:2$ (2) $1:\sqrt{3}:\sqrt{2}$
(3) $\sqrt{2}:\sqrt{3}:1$ (4) $\sqrt{3}:\sqrt{2}:1$

Ans. (1)

Sol. $KE = \frac{P^2}{2m}$

$P \propto \sqrt{m}$

Hence, $P_A : P_B : P_C$

$= \sqrt{400} : \sqrt{1200} : \sqrt{1600} = 1:\sqrt{3}:2$

32. Average force exerted on a non-reflecting surface at normal incidence is 2.4×10^{-4} N. If 360 W/cm^2 is the light energy flux during span of 1 hour 30 minutes. Then the area of the surface is:

- (1) 0.2 m^2 (2) 0.02 m^2
(3) 20 m^2 (4) 0.1 m^2

Ans. (2)

Sol. Pressure = $\frac{I}{C} = \frac{F}{A}$

$\Rightarrow \frac{360}{10^{-4} \times 3 \times 10^8} = \frac{2.4 \times 10^{-4}}{A}$

$\Rightarrow A = 2 \times 10^{-2} \text{ m}^2 = 0.02 \text{ m}^2$

33. A proton and an electron are associated with same de-Broglie wavelength. The ratio of their kinetic energies is:

(Assume $h = 6.63 \times 10^{-34} \text{ J s}$, $m_e = 9.0 \times 10^{-31} \text{ kg}$ and $m_p = 1836 \text{ times } m_e$)

- (1) $1 : 1836$ (2) $1 : \frac{1}{1836}$
(3) $1 : \frac{1}{\sqrt{1836}}$ (4) $1 : \sqrt{1836}$

TEST PAPER WITH SOLUTION

Ans. (1)

Sol. λ is same for both

$P = \frac{h}{\lambda}$ same for both

$P = \sqrt{2mK}$

Hence,

$K \propto \frac{1}{m}$

$\Rightarrow \frac{KE_p}{KE_e} = \frac{m_e}{m_p} = \frac{1}{1836}$

34. A mixture of one mole of monoatomic gas and one mole of a diatomic gas (rigid) are kept at room temperature (27°C). The ratio of specific heat of gases at constant volume respectively is:

- (1) $\frac{7}{5}$ (2) $\frac{3}{2}$
(3) $\frac{3}{5}$ (4) $\frac{5}{3}$

Ans. (3)

Sol. $\frac{(C_v)_{\text{mono}}}{(C_v)_{\text{dia}}} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}$

35. In an expression $a \times 10^b$:

- (1) a is order of magnitude for $b \leq 5$
(2) b is order of magnitude for $a \leq 5$
(3) b is order of magnitude for $5 < a \leq 10$
(4) b is order of magnitude for $a \geq 5$

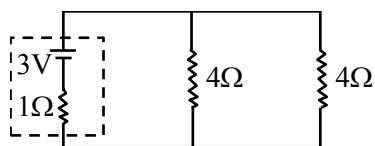
Ans. (2)

Sol. $a \times 10^b$

if $a \leq 5$ order is b

$a > 5$ order is $b + 1$

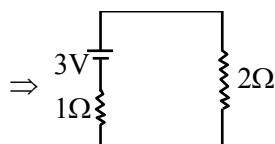
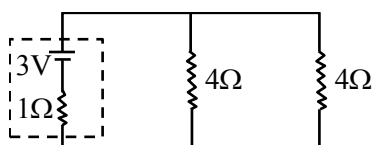
36. In the given circuit, the terminal potential difference of the cell is :



- (1) 2 V (2) 4 V
(3) 1.5 V (4) 3 V

Ans. (1)

Sol.



$$i = \frac{3}{1+2} = 1A$$

$$v = E - ir$$

$$= 3 - 1 \times 1 = 2V$$

37. Binding energy of a certain nucleus is 18×10^8 J. How much is the difference between total mass of all the nucleons and nuclear mass of the given nucleus:

- (1) 0.2 μ g (2) 20 μ g
(3) 2 μ g (4) 10 μ g

Ans. (2)

Sol. $\Delta mc^2 = 18 \times 10^8$

$$\Delta m \times 9 \times 10^{16} = 18 \times 10^8$$

$$\Delta m = 2 \times 10^{-8} \text{ kg} = 20 \mu\text{g}$$

38. Paramagnetic substances:

- A. align themselves along the directions of external magnetic field.
B. attract strongly towards external magnetic field.
C. has susceptibility little more than zero.
D. move from a region of strong magnetic field to weak magnetic field.

Choose the **most appropriate** answer from the options given below:

- (1) A, B, C, D (2) B, D Only
(3) A, B, C Only (4) A, C Only

Ans. (4)

Sol. A, C only

39. A clock has 75 cm, 60 cm long second hand and minute hand respectively. In 30 minutes duration the tip of second hand will travel x distance more than the tip of minute hand. The value of x in meter is nearly (Take $\pi = 3.14$) :

- (1) 139.4 (2) 140.5
(3) 220.0 (4) 118.9

Ans. (1)

Sol. $x_{\min} = \pi \times r_{\min}$

$$= \pi \times \frac{60}{100} \text{ m.}$$

$$x_{\text{second}} = 30 \times 2\pi \times r_{\text{second}}$$

$$= 30 \times 2\pi \times \frac{75}{100}$$

$$x = x_{\text{second}} - x_{\min}$$

$$= 139.4 \text{ m}$$

40. Young's modulus is determined by the equation

given by $Y = 49000 \frac{\text{m dyne}}{\ell \text{ cm}^2}$ where M is the mass

and ℓ is the extension of wire used in the experiment. Now error in Young modules(Y) is estimated by taking data from M- ℓ plot in graph paper. The smallest scale divisions are 5 g and 0.02 cm along load axis and extension axis respectively. If the value of M and ℓ are 500 g and 2 cm respectively then percentage error of Y is :

- (1) 0.2 % (2) 0.02 %
(3) 2 % (4) 0.5 %

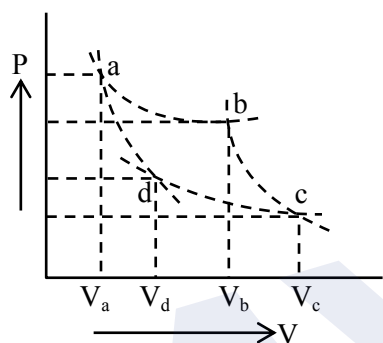
Ans. (3)

Sol. $\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta \ell}{\ell}$
 $= \frac{5}{500} + \frac{0.02}{2} = 0.01 + 0.01$

$\frac{\Delta Y}{Y} = 0.02 \Rightarrow \% \frac{\Delta Y}{Y} = 2\%$

- 41.** Two different adiabatic paths for the same gas intersect two isothermal curves as shown in P-V diagram. The relation between the ratio $\frac{V_a}{V_d}$ and the

ratio $\frac{V_b}{V_c}$ is:



- (1) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{-1}$ (2) $\frac{V_a}{V_d} \neq \frac{V_b}{V_c}$
 (3) $\frac{V_a}{V_d} = \frac{V_b}{V_c}$ (4) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^2$

Ans. (3)

Sol. For adiabatic process

$TV^{\gamma-1} = \text{constant}$

$T_a \cdot V_a^{\gamma-1} = T_d \cdot V_d^{\gamma-1}$

$\left(\frac{V_a}{V_d}\right)^{\gamma-1} = \frac{T_d}{T_a}$

$T_b \cdot V_b^{\gamma-1} = T_c \cdot V_c^{\gamma-1}$

$\left(\frac{V_b}{V_c}\right)^{\gamma-1} = \frac{T_c}{T_b}$

$\frac{V_a}{V_d} = \frac{V_b}{V_c} \quad \left(\because \begin{matrix} T_d = T_c \\ T_a = T_b \end{matrix} \right)$

- 42.** Two planets A and B having masses m_1 and m_2 move around the sun in circular orbits of r_1 and r_2 radii respectively. If angular momentum of A is L and that of B is $3L$, the ratio of time period $\left(\frac{T_A}{T_B}\right)$ is:

- (1) $\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$ (2) $\left(\frac{r_1}{r_2}\right)^3$
 (3) $\frac{1}{27} \left(\frac{m_2}{m_1}\right)^3$ (4) $27 \left(\frac{m_1}{m_2}\right)^3$

Ans. (3)

Sol. $\frac{\pi r_1^2}{T_A} = \frac{L}{2m_1} \dots\dots (1)$

$\frac{\pi r_2^2}{T_B} = \frac{3L}{2m_2} \dots\dots (2)$

$\Rightarrow \frac{T_A}{T_B} = 3 \cdot \frac{m_1}{m_2} \cdot \left(\frac{r_1}{r_2}\right)^2$

$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{T_A}{T_B}\right)^{\frac{4}{3}}$

$\Rightarrow \frac{1}{27} \cdot \left(\frac{m_2}{m_1}\right)^3 = \left(\frac{T_A}{T_B}\right)$

- 43.** A LCR circuit is at resonance for a capacitor C , inductance L and resistance R . Now the value of resistance is halved keeping all other parameters same. The current amplitude at resonance will be now:

- (1) Zero (2) double
 (3) same (4) halved

Ans. (2)

Sol. In resonance $Z = R$

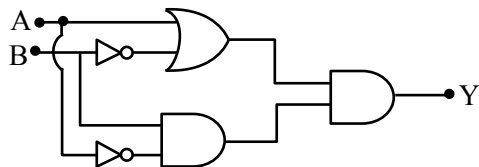
$I = \frac{V}{R}$

$R \rightarrow \text{halved}$

$\Rightarrow I \rightarrow 2I$

I becomes doubled.

44. The output Y of following circuit for given inputs is :



- (1) $A \cdot B(A + B)$ (2) $A \cdot B$
(3) 0 (4) $\bar{A} \cdot B$

Ans. (3)

Sol. By truth table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

45. Two charged conducting spheres of radii a and b are connected to each other by a conducting wire. The ratio of charges of the two spheres respectively is:

- (1) \sqrt{ab} (2) ab
(3) $\frac{a}{b}$ (4) $\frac{b}{a}$

Ans. (3)

Sol. Potential at surface will be same

$$\frac{Kq_1}{a} = \frac{Kq_2}{b}$$

$$\frac{q_1}{q_2} = \frac{a}{b}$$

46. Correct Bernoulli's equation is (symbols have their usual meaning) :

- (1) $P + mgh + \frac{1}{2}mv^2 = \text{constant}$
(2) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$
(3) $P + \rho gh + \rho v^2 = \text{constant}$
(4) $P + \frac{1}{2}\rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Ans. (2)

Sol. $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

47. A player caught a cricket ball of mass 150 g moving at a speed of 20 m/s. If the catching process is completed in 0.1 s, the magnitude of force exerted by the ball on the hand of the player is:

- (1) 150 N (2) 3 N
(3) 30 N (4) 300 N

Ans. (3)

Sol. $F = \frac{\Delta P}{\Delta t} = \frac{mv - 0}{0.1}$
 $= \frac{150 \times 10^{-3} \times 20}{0.1} = 30 \text{ N}$

48. A stationary particle breaks into two parts of masses m_A and m_B which move with velocities v_A and v_B respectively. The ratio of their kinetic energies ($K_B : K_A$) is :

- (1) $v_B : v_A$ (2) $m_B : m_A$
(3) $m_B v_B : m_A v_A$ (4) 1 : 1

Ans. (1)

Sol. Initial momentum is zero.

Hence $|P_A| = |P_B|$

$\Rightarrow m_A v_A = m_B v_B$

$\frac{(KE)_A}{(KE)_B} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{v_A}{v_B}$

$\frac{(KE)_B}{(KE)_A} = \frac{v_B}{v_A}$

49. Critical angle of incidence for a pair of optical media is 45° . The refractive indices of first and second media are in the ratio:

- (1) $\sqrt{2} : 1$ (2) 1 : 2
(3) $1 : \sqrt{2}$ (4) 2 : 1

Ans. (1)

Sol. $\sin \theta_c = \frac{\mu_R}{\mu_d} = \frac{\mu_2}{\mu_1}$

$$\sin 45^\circ = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{\sqrt{2}}{1}$$

- 50.** The diameter of a sphere is measured using a vernier caliper whose 9 divisions of main scale are equal to 10 divisions of vernier scale. The shortest division on the main scale is equal to 1 mm. The main scale reading is 2 cm and second division of vernier scale coincides with a division on main scale. If mass of the sphere is 8.635 g, the density of the sphere is:

(1) 2.5 g/cm³ (2) 1.7 g/cm³

(3) 2.2 g/cm³ (4) 2.0 g/cm³

Ans. (4)

Sol. Given 9MSD = 10VSD

$$\text{mass} = 8.635 \text{ g}$$

$$\text{LC} = 1 \text{ MSD} - 1 \text{ VSD}$$

$$\text{LC} = 1 \text{ MSD} - \frac{9}{10} \text{ MSD}$$

$$\text{LC} = \frac{1}{10} \text{ MSD}$$

$$\text{LC} = 0.01 \text{ cm}$$

$$\text{Reading of diameter} = \text{MSR} + \text{LC} \times \text{VSR}$$

$$= 2 \text{ cm} + (0.01) \times (2)$$

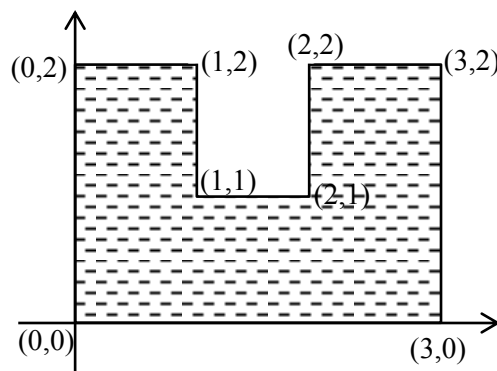
$$= 2.02 \text{ cm}$$

$$\begin{aligned} \text{Volume of sphere} &= \frac{4}{3} \pi \left(\frac{d}{2} \right)^3 = \frac{4}{3} \pi \left(\frac{2.02}{2} \right)^3 \\ &= 4.32 \text{ cm}^3 \end{aligned}$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{8.635}{4.32} = 1.998 \sim 2.00 \text{ g}$$

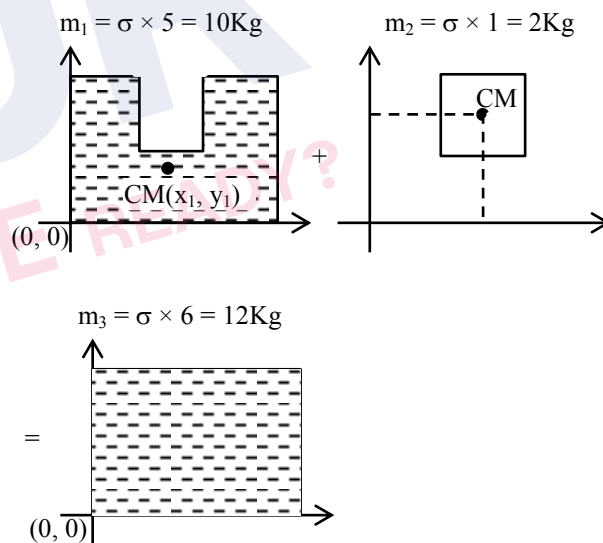
SECTION-B

- 51.** A uniform thin metal plate of mass 10 kg with dimensions is shown. The ratio of x and y coordinates of center of mass of plate in $\frac{n}{9}$. The value of n is _____.



Ans. (15)

Sol. $m_1 = \sigma \times 5 = 10 \text{ Kg}$



$$\Rightarrow m_1 x_1 + m_2 x_2 = m_3 x_3$$

$$10x_1 + 2(1.5) = 12(1.5) \Rightarrow x_1 = 1.5 \text{ cm}$$

$$\Rightarrow m_1 y_1 + m_2 y_2 = m_3 y_3$$

$$10y_1 + 2(1.5) = 12 \times 1 \Rightarrow y_1 = 0.9 \text{ cm}$$

$$\frac{x_1}{y_1} = \frac{1.5}{0.9} = \frac{15}{9}$$

$$n = 15$$

52. An electron with kinetic energy 5 eV enters a region of uniform magnetic field of $3 \mu\text{T}$ perpendicular to its direction. An electric field E is applied perpendicular to the direction of velocity and magnetic field. The value of E , so that electron moves along the same path, is _____ NC^{-1} .

(Given, mass of electron = $9 \times 10^{-31} \text{ kg}$, electric charge = $1.6 \times 10^{-19} \text{ C}$)

Ans. (4)

Sol. For the given condition of moving undeflected, net force should be zero.

$$qE = qVB$$

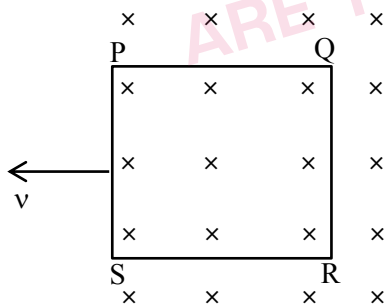
$$E = VB$$

$$= \sqrt{\frac{2 \times KE}{m}} \times B$$

$$= \sqrt{\frac{2 \times 5 \times 1.6 \times 10^{-19}}{9 \times 10^{-31}}} \times 3 \times 10^{-6}$$

$$= 4 \text{ N/C}$$

53. A square loop PQRS having 10 turns, area $3.6 \times 10^{-3} \text{ m}^2$ and resistance 100Ω is slowly and uniformly being pulled out of a uniform magnetic field of magnitude $B = 0.5 \text{ T}$ as shown. Work done in pulling the loop out of the field in 1.0 s is _____ $\times 10^{-6} \text{ J}$.



Ans. (3)

Sol. $\epsilon = NB\ell v$

$$i = \frac{\epsilon}{R} = \frac{NB\ell v}{R}$$

$$F = N(i\ell B) = \frac{N^2 B^2 \ell^2 v}{R}$$

$$W = F \times \ell = \frac{N^2 B^2 \ell^3}{R} \left(\frac{\ell}{t} \right)$$

$$A = \ell^2$$

$$W = \frac{(10 \times 10)(0.5)^2 \times (3.6 \times 10^{-3})^2}{100 \times 1}$$

$$W = 3.24 \times 10^{-6} \text{ J}$$

54. Resistance of a wire at 0°C , 100°C and $t^\circ\text{C}$ is found to be 10Ω , 10.2Ω and 10.95Ω respectively. The temperature t in Kelvin scale is _____.

Ans. (748)

Sol. $R = R_0(1 + \alpha\Delta T)$

$$\frac{\Delta R}{R_0} = \alpha\Delta T$$

Case-I

$$0^\circ\text{C} \rightarrow 100^\circ\text{C}$$

$$\frac{10.2 - 10}{10} = \alpha(100 - 0) \quad \dots (1)$$

Case-II

$$0^\circ\text{C} \rightarrow t^\circ\text{C}$$

$$\frac{10.95 - 10}{10} = \alpha(t - 0) \quad \dots (2)$$

$$\Rightarrow \frac{t}{100} = \frac{0.95}{0.2} = 475^\circ\text{C}$$

$$t = 475 + 273 = 748 \text{ K}$$

55. An electric field, $\vec{E} = \frac{2\hat{i} + 6\hat{j} + 8\hat{k}}{\sqrt{6}}$ passes through the surface of 4 m^2 area having unit vector $\hat{n} = \left(\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}} \right)$. The electric flux for that surface is _____ V m .

Ans. (12)

Sol. $\phi = \vec{E} \cdot \vec{A}$

$$= \left(\frac{2\hat{i} + 6\hat{j} + 8\hat{k}}{\sqrt{6}} \right) \cdot 4 \left(\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}} \right)$$

$$= \frac{4}{6} \times (4 + 6 + 8) = 12 \text{ Vm}$$

56. A liquid column of height 0.04 cm balances excess pressure of soap bubble of certain radius. If density of liquid is $8 \times 10^3 \text{ kg m}^{-3}$ and surface tension of soap solution is 0.28 Nm^{-1} , then diameter of the soap bubble is _____ cm.
(if $g = 10 \text{ ms}^{-2}$)

Ans. (7)

Sol. $\rho gh = \frac{4S}{R}$

$$\Rightarrow R = \frac{4 \times 0.28}{8 \times 10^3 \times 10 \times 4 \times 10^{-4}}$$

$$\Rightarrow \frac{0.28}{8} \text{ m} = \frac{28}{8} \text{ cm}$$

$$\Rightarrow R = 3.5 \text{ cm}$$

$$\text{Diameter} = 7 \text{ cm}$$

57. A closed and an open organ pipe have same lengths. If the ratio of frequencies of their seventh overtones is $\left(\frac{a-1}{a}\right)$ then the value of a is _____.

Ans. (16)

Sol. For closed organ pipe

$$f_c = (2n+1) \frac{v}{4\ell} = \frac{15v}{4\ell}$$

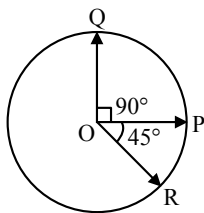
For open organ pipe

$$f_o = (n+1) \frac{v}{2\ell} = \frac{8v}{2\ell}$$

$$\frac{f_c}{f_o} = \frac{15}{16} = \frac{a-1}{a}$$

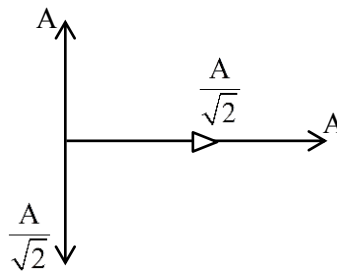
$$\Rightarrow a = 16$$

58. Three vectors \vec{OP} , \vec{OQ} and \vec{OR} each of magnitude A are acting as shown in figure. The resultant of the three vectors is $A\sqrt{x}$. The value of x is _____.



Ans. (3)

Sol.



$$\vec{R} = \left(A + \frac{A}{\sqrt{2}}\right)\hat{i} + \left(A - \frac{A}{\sqrt{2}}\right)\hat{j}$$

$$|\vec{R}| = \sqrt{\left(A + \frac{A}{\sqrt{2}}\right)^2 + \left(A - \frac{A}{\sqrt{2}}\right)^2} = \sqrt{3}A$$

59. A parallel beam of monochromatic light of wavelength 600 nm passes through single slit of 0.4 mm width. Angular divergence corresponding to second order minima would be _____ $\times 10^{-3}$ rad.

Ans. (6)

Sol. $\sin \theta \approx \theta = \frac{2\lambda}{b}$

$$= \frac{2 \times 600 \times 10^{-9}}{4 \times 10^{-4}} = 3 \times 10^{-3} \text{ rad}$$

$$\text{Total divergence} = (3 + 3) \times 10^{-3} = 6 \times 10^{-3} \text{ rad}$$

60. In an alpha particle scattering experiment distance of closest approach for the α particle is $4.5 \times 10^{-14} \text{ m}$. If target nucleus has atomic number 80, then maximum velocity of α -particle is _____ $\times 10^5 \text{ m/s}$ approximately.

$$\left(\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ SI unit, mass of } \alpha \text{ particle} = \right.$$

$$6.72 \times 10^{-27} \text{ kg})$$

Ans. (156)

Sol. $v = \sqrt{\frac{4KZe^2}{mr_{\min}}}$

$$= \sqrt{\frac{4 \times 9 \times 10^9 \times 80}{6.72 \times 10^{-27} \times 4.5 \times 10^{-14}}} \times 1.6 \times 10^{-19}$$

$$= 9.759 \times 10^{25} \times 1.6 \times 10^{-19}$$

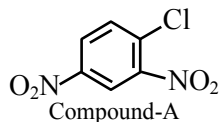
$$= 156 \times 10^5 \text{ m/s}$$

CHEMISTRY

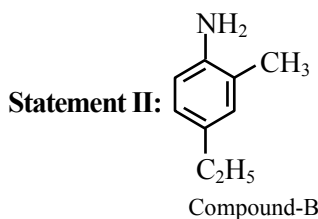
SECTION-A

61. Given below are two statements:

Statement I :



IUPAC name of Compound A is 4-chloro-1,3-dinitrobenzene:



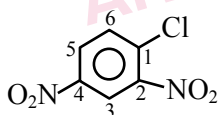
IUPAC name of Compound B is 4-ethyl-2-methylaniline.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (1) Both Statement I and Statement II are correct
- (2) Statement I is incorrect but Statement II is correct
- (3) Statement I is correct but Statement II is incorrect
- (4) Both Statement I and Statement II are incorrect

Ans. (2)

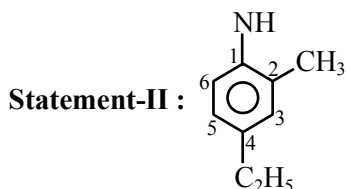
Sol. Statement I :



IUPAC name

⇒ 1-chloro-2,4-dinitrobenzene

⇒ statement-I is incorrect



⇒ 4-ethyl-2-methylaniline

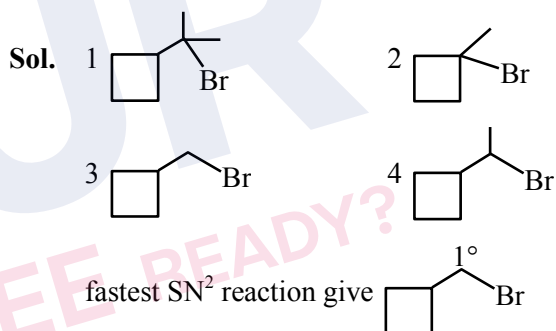
⇒ statement-II is correct

TEST PAPER WITH SOLUTION

62. Which among the following compounds will undergo fastest S_N2 reaction.

- (1)
- (2)
- (3)
- (4)

Ans. (3)



Rate of S_N2 is $Me-x > 1^\circ-x > 2^\circ-x > 3^\circ-x$

63. Combustion of glucose ($C_6H_{12}O_6$) produces CO_2 and water. The amount of oxygen (in g) required for the complete combustion of 900 g of glucose is:
[Molar mass of glucose in $g\ mol^{-1} = 180$]

- (1) 480
- (2) 960
- (3) 800
- (4) 32

Ans. (2)

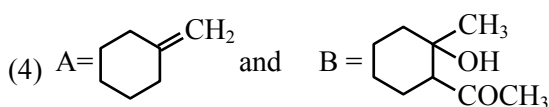
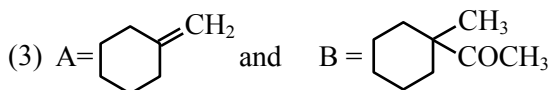
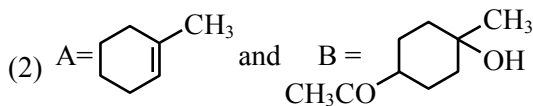
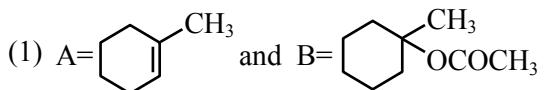
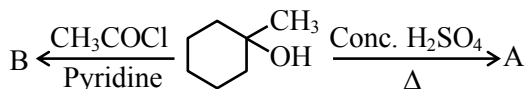
Sol. $C_6H_{12}O_{6(s)} + 6O_{2(g)} \longrightarrow 6CO_{2(g)} + 6H_2O_{(l)}$

$$\frac{900}{180}$$

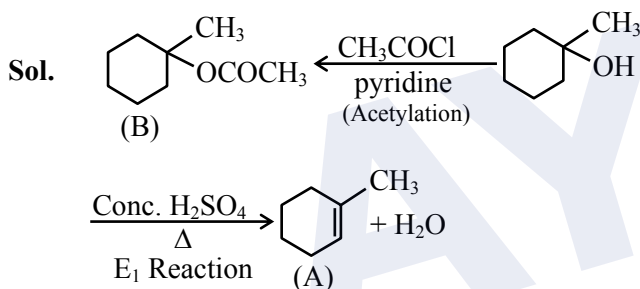
$$= 5\ mol \quad 30\ mol$$

$$\text{Mass of } O_2 \text{ required} = 30 \times 32 = 960\ gm$$

64. Identify the major products A and B respectively in the following set of reactions.



Ans. (1)



65. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A : The stability order of +1 oxidation state of Ga, In and Tl is $\text{Ga} < \text{In} < \text{Tl}$.

Reason R : The inert pair effect stabilizes the lower oxidation state down the group.

In the light of the above statements, choose the correct answer from the options given below :

- (1) Both **A** and **R** are true and **R** is the correct explanation of **A**.
- (2) **A** is true but **R** is false.
- (3) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.
- (4) **A** is false but **R** is true.

Ans. (1)

Sol. The relative stability of +1 oxidation state progressively increases for heavier elements due to inert pair effect.

$$\therefore \text{Stability of } \text{Al}^{+1} < \text{Ga}^{+1} < \text{In}^{+1} < \text{Tl}^{+1}$$

66. Match List I with List-II

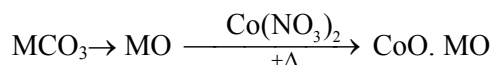
List-I (Name of the test)		List-II (Reaction sequence involved) [M is metal]	
A	Borax bead test	I.	$\text{MCO}_3 \rightarrow \text{MO}$ $\xrightarrow{+\Delta} \text{Co(NO}_3)_2 \rightarrow \text{CoO} \cdot \text{MO}$
B.	Charcoal cavity test	II.	$\text{MCO}_3 \rightarrow \text{MCl}_2 \rightarrow \text{M}^{2+}$
C.	Cobalt nitrate test	III.	$\text{MSO}_4 \xrightarrow[\Delta]{\text{Na}_2\text{B}_4\text{O}_7}$ $\text{M(BO}_2)_2 \rightarrow \text{MBO}_2 \rightarrow \text{M}$
D.	Flame test	IV.	$\text{MSO}_4 \xrightarrow[\Delta]{\text{Na}_2\text{CO}_3} \text{MCO}_3 \rightarrow$ $\text{MO} \rightarrow \text{M}$

Choose the correct answer from the option below :

- (1) A-III, B-I, C-IV, D-II
- (2) A-III, B-II, C-IV, D-I
- (3) A-III, B-I, C-II, D-IV
- (4) A-III, B-IV, C-I, D-II

Ans. (4)

Sol. **Cobalt nitrate test**



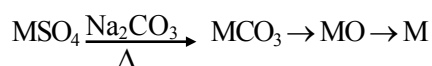
Flame test



Borax Bead test



Charcoal cavity test



67. Match List I and with List II

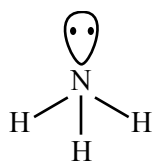
List-I (Molecule)		List-II(Shape)	
A	NH ₃	I.	Square pyramid
B.	BrF ₅	II.	Tetrahedral
C.	PCl ₅	III	Trigonal pyramidal
D.	CH ₄	IV	Trigonal bipyramidal

Choose the **correct** answer from the option below :

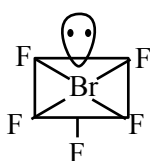
- (1) A-IV, B-III, C-I, D-II
(2) A-II, B-IV, C-I, D-III
(3) A-III, B-I, C-IV, D-II
(4) A-III, B-IV, C-I, D-II

Ans. (3)

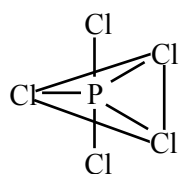
Sol.



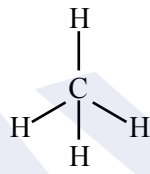
Trigonal pyramidal



Square pyramidal

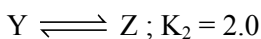


Trigonal bipyramidal

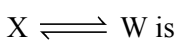


Tetrahedral

68. For the given hypothetical reactions, the equilibrium constants are as follows:

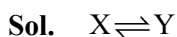


The equilibrium constant for the reaction



- (1) 6.0 (2) 12.0
(3) 8.0 (4) 7.0

Ans. (3)



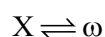
$$k_1 = 1$$



$$k_2 = 2$$



$$k_3 = 4$$

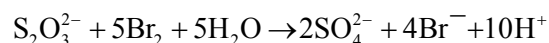
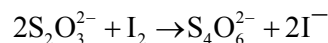


$$k_1 \cdot k_2 \cdot k_3$$

$$k = 1 \times 2 \times 4$$

$$k = 8$$

69. Thiosulphate reacts differently with iodine and bromine in the reaction given below :



Which of the following statement justifies the above dual behaviour of thiosulphate?

- (1) Bromine undergoes oxidation and iodine undergoes reduction by iodine in these reactions
(2) Thiosulphate undergoes oxidation by bromine and reduction by iodine in these reaction
(3) Bromine is a stronger oxidant than iodine
(4) Bromine is a weaker oxidant than iodine

Ans. (3)

Sol. In the reaction of $S_2O_3^{2-}$ with I_2 , oxidation state of sulphur changes to +2 to +2.5

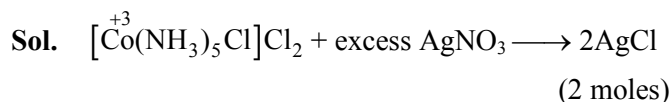
In the reaction of $S_2O_3^{2-}$ with Br_2 , oxidation state of sulphur changes from +2 to +6.

\therefore Both I_2 and Br_2 are oxidant (oxidising agent) and Br_2 is stronger oxidant than I_2 .

70. An octahedral complex with the formula $CoCl_3nNH_3$ upon reaction with excess of $AgNO_3$ solution given 2 moles of $AgCl$. Consider the oxidation state of Co in the complex is 'x'. The value of "x + n" is _____.

- (1) 3 (2) 6
(3) 8 (4) 5

Ans. (3)

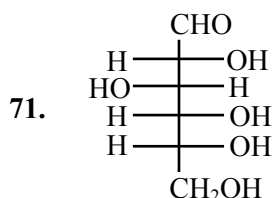


$$x + 0 - 1 - 2 = 0$$

$$x = +3$$

$$n = 5$$

$$\therefore x + n = 8$$

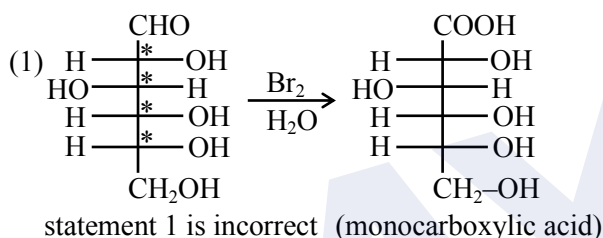


The **incorrect** statement regarding the given structure is

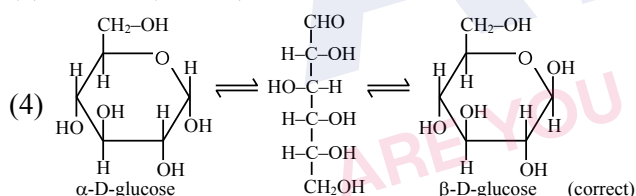
- (1) Can be oxidized to a dicarboxylic acid with Br_2 water
- (2) despite the presence of $-\text{CHO}$ does not give Schiff's test
- (3) has 4-asymmetric carbon atom
- (4) will coexist in equilibrium with 2 other cyclic structure

Ans. (1)

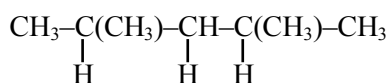
Sol.



- (2) correct
- (3) c.c. is 4 (correct)

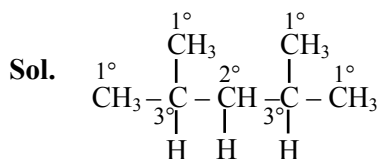


72. In the given compound, the number of 2° carbon atom/s is _____.



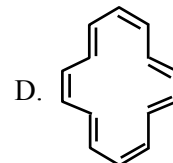
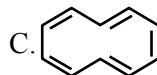
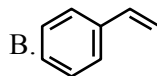
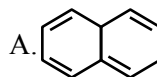
- (1) Three
- (2) One
- (3) Two
- (4) Four

Ans. (2)



only one 2° carbon is present in this compound.

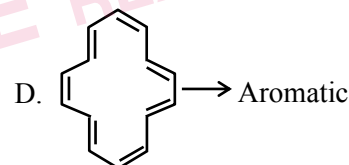
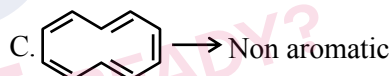
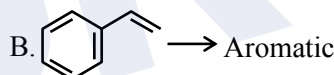
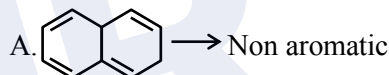
73. Which of the following are aromatic?



- (1) B and D only
- (2) A and C only
- (3) A and B only
- (4) C and D only

Ans. (1)

Sol.



74. Among the following halogens

F_2 , Cl_2 , Br_2 and I_2

Which can undergo disproportionation reaction?

- (1) Only I_2
- (2) Cl_2 , Br_2 and I_2
- (3) F_2 , Cl_2 and Br_2
- (4) F_2 and Cl_2

Ans. (2)

Sol. F_2 do not disproportionate because fluorine do not exist in positive oxidation state however Cl_2 , Br_2 & I_2 undergoes disproportionation.

75. Given below are two statements:

Statement I : $N(CH_3)_3$ and $P(CH_3)_3$ can act as ligands to form transition metal complexes.

Statement II: As N and P are from same group, the nature of bonding of $N(CH_3)_3$ and $P(CH_3)_3$ is always same with transition metals.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (1) Statement I is incorrect but Statement II is correct
- (2) Both Statement I and Statement II are correct
- (3) Statement I is correct but Statement II is incorrect
- (4) Both Statement I and Statement II are incorrect

Ans. (3)

Sol. $N(CH_3)_3$ and $P(CH_3)_3$ both are Lewis base and acts as ligand, However, $P(CH_3)_3$ has a π -acceptor character.

76. Match **List I** with **List II**

List-I (Elements)		List-II (Properties in their respective groups)	
A	Cl, S	I.	Elements with highest electronegativity
B.	Ge, As	II.	Elements with largest atomic size
C.	Fr, Ra	III	Elements which show properties of both metals and non metal
D.	F, O	IV	Elements with highest negative electron gain enthalpy

Choose the **correct** answer from the options given below :

- (1) A-II, B-III, C-IV, D-I
- (2) A-III, B-II, C-I, D-IV
- (3) A-IV, B-III, C-II, D-I
- (4) A-II, B-I, C-IV, D-III

Ans. (3)

Sol. Elements with highest electronegativity \rightarrow F, O

Elements with largest atomic size \rightarrow Fr, Ra

Elements which shows properties of both metal and non-metals i.e. metalloids \rightarrow Ge, As

Elements with highest negative electron gain enthalpy \rightarrow Cl, S

77. Iron (III) catalyses the reaction between iodide and persulphate ions, in which

- A. Fe^{3+} oxidises the iodide ion
- B. Fe^{3+} oxidises the persulphate ion
- C. Fe^{2+} reduces the iodide ion
- D. Fe^{2+} reduces the persulphate ion

Choose the **most appropriate** answer from the options given below:

- (1) B and C only
- (2) B only
- (3) A only
- (4) A and D only

Ans. (4)

Sol. $2Fe^{3+} + 2I^- \longrightarrow 2Fe^{2+} + I_2$



Fe^{3+} oxidises I^- to I_2 and convert itself into Fe^{2+} . This Fe^{2+} reduces $S_2O_8^{2-}$ to SO_4^{2-} and converts itself into Fe^{3+} .

78. Match **List I** with **List II**

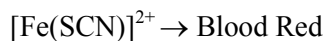
List-I (Compound)		List-II (Colour)	
A	$Fe_4[Fe(CN)_6]_3 \cdot xH_2O$	I.	Violet
B.	$[Fe(CN)_5NOS]^{4-}$	II.	Blood Red
C.	$[Fe(SCN)]^{2+}$	III.	Prussian Blue
D.	$(NH_4)_3PO_4 \cdot 12MoO_3$	IV.	Yellow

Choose the **correct** answer from the options given below :

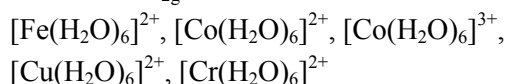
- (1) A-III, B-I, C-II, D-IV
- (2) A-IV, B-I, C-II, D-III
- (3) A-II, B-III, C-IV, D-I
- (4) A-I, B-II, C-III, D-IV

Ans. (1)

Sol. $Fe_4[Fe(CN)_6]_3 \cdot xH_2O \rightarrow$ Prussian Blue



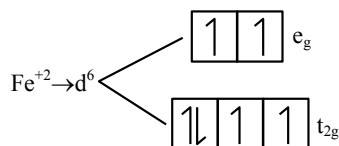
79. Number of complexes with even number of electrons in t_{2g} orbitals is -



- (1) 1
- (2) 3
- (3) 2
- (4) 5

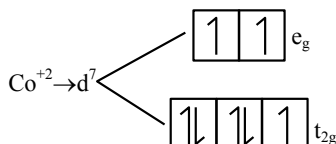
Ans. (2)

Sol. $[\text{Fe}(\text{H}_2\text{O})_6]^{2+}$



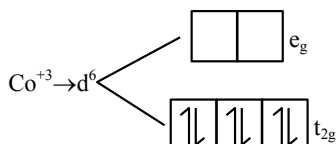
Electron in $t_{2g} = 4(\text{even})$

$[\text{Co}(\text{H}_2\text{O})_6]^{2+}$



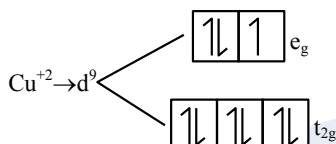
Electron in $t_{2g} = 5(\text{odd})$

$[\text{Co}(\text{H}_2\text{O})_6]^{3+}$



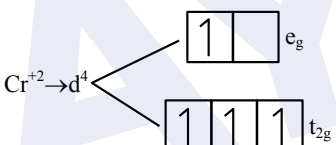
Electron in $t_{2g} = 6(\text{even})$

$[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$



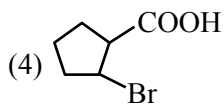
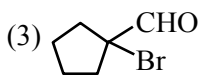
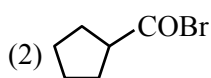
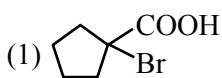
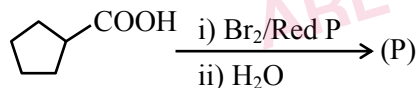
Electron in $t_{2g} = 6(\text{even})$

$[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$



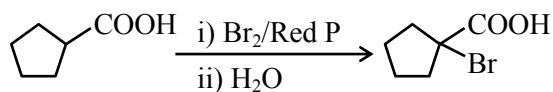
Electron in $t_{2g} = 3(\text{odd})$

80. Identify the product (P) in the following reaction:



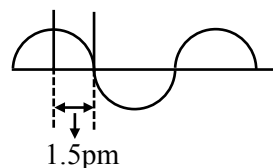
Ans. (1)

Sol. HVZ Reaction



SECTION-B

81. A hypothetical electromagnetic wave is shown below.



The frequency of the wave is $x \times 10^{19}$ Hz.

$x = \underline{\hspace{2cm}}$ (nearest integer)

Ans. (5)

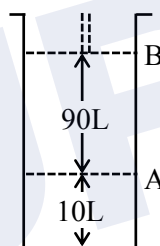
Sol. $\lambda = 1.5 \times 4 \text{ pm}$
 $= 6 \times 10^{-12} \text{ meter}$

$$\lambda \nu = C$$

$$6 \times 10^{-12} \times \nu = 3 \times 10^8$$

$$\nu = 5 \times 10^{19} \text{ Hz}$$

82.



Consider the figure provided.

1 mol of an ideal gas is kept in a cylinder, fitted with a piston, at the position A, at 18°C . If the piston is moved to position B, keeping the temperature unchanged, then 'x' L atm work is done in this reversible process.

$x = \underline{\hspace{2cm}}$ L atm. (nearest integer)

[Given : Absolute temperature = $^\circ\text{C} + 273.15$,
 $R = 0.08206 \text{ L atm mol}^{-1} \text{ K}^{-1}$]

Ans. (55)

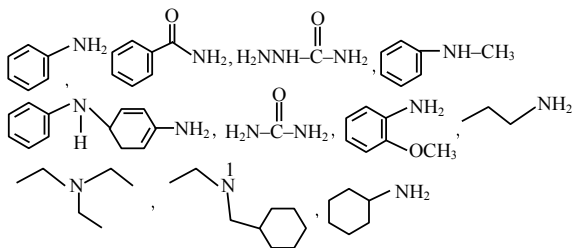
Sol. $\omega = -nRT \ln \left(\frac{V_2}{V_1} \right)$

$$= -1 \times 0.08206 \times 291.15 \ln \left(\frac{100}{10} \right)$$

$$= -55.0128$$

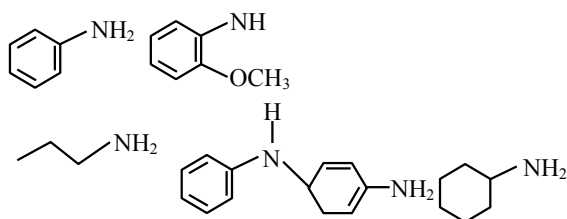
Work done by system $\approx 55 \text{ atm lit.}$

83. Number of amine compounds from the following giving solids which are soluble in NaOH upon reaction with Hinsberg's reagent is ____.

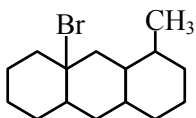


Ans. (5)

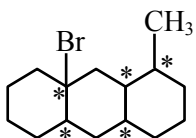
Sol. Primary amine give an ionic solid upon reaction with Hinsberg reagent which is soluble in NaOH.



84. The number of optical isomers in following compound is : _____.



Ans. (32)



Sol.

Total chiral centre = 5

No. of optical isomers = $2^5 = 32$.

- 85.** The 'spin only' magnetic moment value of MO_4^{2-} is ____ BM. (Where M is a metal having least metallic radii. among Sc, Ti, V, Cr, Mn and Zn).
(Given atomic number : Sc = 21, Ti = 22, V = 23, Cr = 24, Mn = 25 and Zn = 30)

Ans. (0)

Sol. Metal having least metallic radii among Sc, Ti, V, Cr, Mn & Zn is Cr.

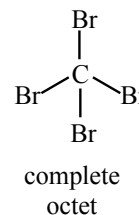
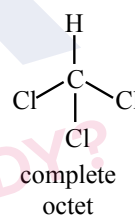
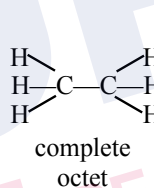
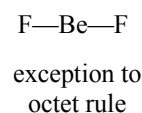
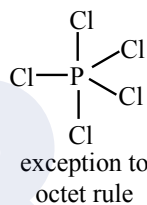
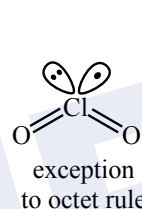
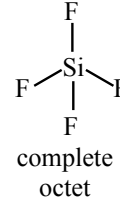
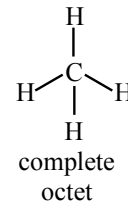
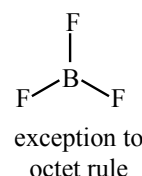
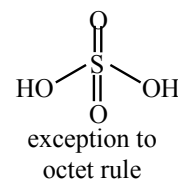
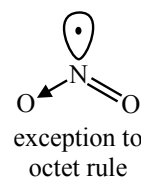
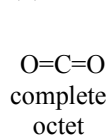
Spin only magnetic moment of CrO_4^{2-} .

Here Cr^{+6} is in d^0 configuration (diamagnetic).

- 86.** Number of molecules from the following which are exceptions to octet rule is _____.
 CO_2 , NO_2 , H_2SO_4 , BF_3 , CH_4 , SiF_4 , ClO_2 , PCl_5 ,
 BeF_2 , C_2H_6 , CHCl_3 , CBr_4

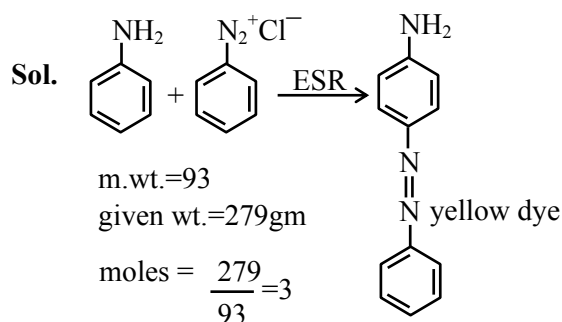
Ans. (6)

Sol.



87. If 279 g of aniline is reacted with one equivalent of benzenediazonium chloride, the maximum amount of aniline yellow formed will be _____ g. (nearest integer)
(consider complete conversion)

Ans. (591)



moles formed = 3

m.wt. = 197

amount formed

$$= 197 \times 3 = 591 \text{ gm}$$

88. Consider the following reaction



The time taken for A to become $1/4^{\text{th}}$ of its initial concentration is twice the time taken to become $1/2$ of the same. Also, when the change of concentration of B is plotted against time, the resulting graph gives a straight line with a negative slope and a positive intercept on the concentration axis.

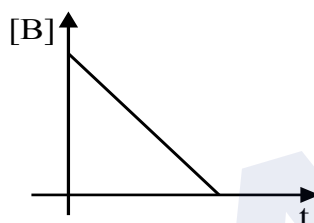
The overall order of the reaction is ____.

Ans. (1)

Sol. For 1^{st} order reaction

$$75\% \text{ life} = 2 \times 50\% \text{ life}$$

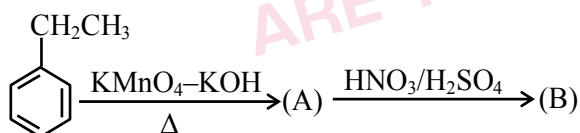
So order with respect to A will be first order.



So order with respect to B will be zero.

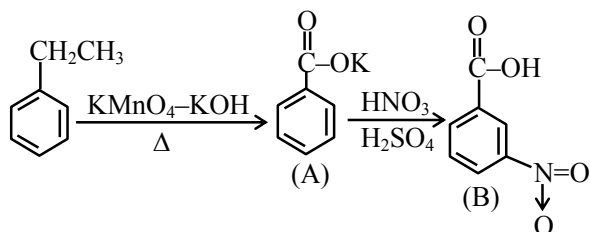
$$\text{Overall order of reaction} = 1 + 0 = 1$$

89. Major product B of the following reaction has ____ π -bond.



Ans. (5)

Sol. Major product B is \rightarrow

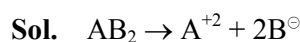


Total number of π bonds in B are 5

90. A solution containing 10g of an electrolyte AB_2 in 100g of water boils at 100.52°C . The degree of ionization of the electrolyte (α) is ____ $\times 10^{-1}$. (nearest integer)

[Given : Molar mass of $\text{AB}_2 = 200\text{g mol}^{-1}$. K_b (molal boiling point elevation const. of water) $= 0.52\text{ K kg mol}^{-1}$, boiling point of water $= 100^\circ\text{C}$; AB_2 ionises as $\text{AB}_2 \rightarrow \text{A}^{2+} + 2\text{B}^-$]

Ans. (5)



$$i = 1 + (3 - 1) \alpha$$

$$i = 1 + 2\alpha$$

$$\Delta T_b = k_b \cdot i \cdot m$$

$$0.52 = 0.52 (1 + 2\alpha) \frac{10}{100} \frac{200}{1000}$$

$$1 = (1 + 2\alpha) \frac{10}{20}$$

$$2 = 1 + 2\alpha$$

$$\alpha = 0.5$$

$$\text{Ans. } \alpha = 5 \times 10^{-1}$$