

# FINAL JEE-MAIN EXAMINATION – APRIL, 2024

(Held On Saturday 06<sup>th</sup> April, 2024)

TIME : 9 : 00 AM to 12 : 00 NOON

## MATHEMATICS

### SECTION-A

1. If  $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ , then

(1)  $f''(0) = 1$  (2)  $f''\left(\frac{2}{\pi}\right) = \frac{24 - \pi^2}{2\pi}$

(3)  $f''\left(\frac{2}{\pi}\right) = \frac{12 - \pi^2}{2\pi}$  (4)  $f''(0) = 0$

Ans. (2)

Sol.  $f(x) = 3x^2 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right)$

$$f'(x) = 6x \sin\left(\frac{1}{x}\right) - 3 \cos\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) - \frac{\sin\left(\frac{1}{x}\right)}{x}$$

$$f''\left(\frac{2}{\pi}\right) = \frac{12}{\pi} - \frac{\pi}{2} = \frac{24 - \pi^2}{2\pi}$$

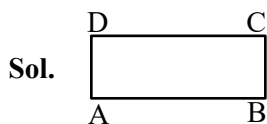
2. If  $A(3, 1, -1)$ ,  $B\left(\frac{5}{3}, \frac{7}{3}, \frac{1}{3}\right)$ ,  $C(2, 2, 1)$  and

$D\left(\frac{10}{3}, \frac{2}{3}, \frac{-1}{3}\right)$  are the vertices of a quadrilateral ABCD, then its area is

(1)  $\frac{4\sqrt{2}}{3}$  (2)  $\frac{5\sqrt{2}}{3}$

(3)  $2\sqrt{2}$  (4)  $\frac{2\sqrt{2}}{3}$

Ans. (1)



$$\text{Area} = \frac{1}{2} |\overrightarrow{BD} \times \overrightarrow{AC}|$$

## TEST PAPER WITH SOLUTION

$$\overrightarrow{BD} = \frac{5}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{2}{3}\hat{k}$$

$$\overrightarrow{AC} = \hat{i} - \hat{j} - 2\hat{k}$$

3.  $\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{(\cos^3 x + \sin^3 x)^2} dx$  is equal to

(1) 1/12

(2) 1/9

(3) 1/6

(4) 1/3

Ans. (3)

Sol. Divide Nr & Dr by  $\cos x$

$$\int_0^{\pi/4} \frac{\tan^2 x \sec^2 x dx}{(1 + \tan^3 x)^2}$$

Let  $1 + \tan^3 x = t$

$$\tan^2 x \sec^2 x dx = \frac{dt}{3}$$

$$\frac{1}{3} \int_1^2 \frac{dt}{t^2} = \frac{1}{6}$$

4. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On respectively, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is

(1)  $\sqrt{3.86}$

(2) 1.8

(3)  $\sqrt{3.96}$

(4) 1.94

Ans. (3)

Sol. Mean  $(\bar{x}) = 10$

$$\Rightarrow \frac{\sum x_i}{20} = 10$$

$$\sum x_i = 10 \times 20 = 200$$

If 8 is replaced by 12, then  $\sum x_i = 200 - 8 + 12 = 204$

$$\therefore \text{Correct mean } (\bar{x}) = \frac{\sum x_i}{20}$$

$$= \frac{204}{20} = 10.2$$

$$\therefore \text{Standard deviation} = 2$$

$$\therefore \text{Variance} = (\text{S.D.})^2 = 2^2 = 4$$

$$\Rightarrow \frac{\sum x_i^2}{20} - \left( \frac{\sum x_i}{20} \right)^2 = 4$$

$$\Rightarrow \frac{\sum x_i^2}{20} - (10)^2 = 4$$

$$\Rightarrow \frac{\sum x_i^2}{20} = 104$$

$$\Rightarrow \sum x_i^2 = 2080$$

Now, replaced '8' observations by '12'

$$\text{Then, } \sum x_i^2 = 2080 - 8^2 + 12^2 = 2160$$

$\therefore$  Variance of removing observations

$$\Rightarrow \frac{\sum x_i^2}{20} - \left( \frac{\sum x_i}{20} \right)^2$$

$$\Rightarrow \frac{2160}{20} - (10.2)^2$$

$$\Rightarrow 108 - 104.04$$

$$\Rightarrow 3.96$$

Correct standard deviation

$$= \sqrt{3.96}$$

5. The function  $f(x) = \frac{x^2 + 2x - 15}{x^2 - 4x + 9}$ ,  $x \in \mathbb{R}$  is

(1) both one-one and onto.

(2) onto but not one-one.

(3) neither one-one nor onto.

(4) one-one but not onto.

**NTA Ans. (3)**

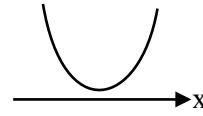
**Ans. Bonus**

**Sol.**  $f(x) = \frac{(x+5)(x-3)}{x^2 - 4x + 9}$

$$\text{Let } g(x) = x^2 - 4x + 9$$

$$D < 0$$

$$g(x) > 0 \text{ for } x \in \mathbb{R}$$



$$\therefore \begin{cases} f(-5) = 0 \\ f(3) = 0 \end{cases}$$

So,  $f(x)$  is many-one.

again,

$$yx^2 - 4xy + 9y = x^2 + 2x - 15$$

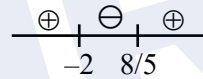
$$x^2(y-1) - 2x(2y+1) + (9y+15) = 0$$

$$\text{for } \forall x \in \mathbb{R} \Rightarrow D \geq 0$$

$$D = 4(2y+1)^2 - 4(y-1)(9y+15) \geq 0$$

$$5y^2 + 2y + 16 \leq 0$$

$$(5y-8)(y+2) \leq 0$$



$$y \in \left[ -2, \frac{8}{5} \right] \text{ range}$$

**Note :** If function is defined from  $f: \mathbb{R} \rightarrow \mathbb{R}$  then only correct answer is option (3)

$\Rightarrow$  Bonus

6. Let  $A = \{n \in [100, 700] \cap \mathbb{N} : n \text{ is neither a multiple of 3 nor a multiple of 4}\}$ . Then the number of elements in  $A$  is

(1) 300

(2) 280

(3) 310

(4) 290

**Ans. (1)**

**Sol.**  $n(3) \Rightarrow$  multiple of 3

$$102, 105, 108, \dots, 699$$

$$T_n = 699 = 102 + (n-1)(3)$$

$$n = 200$$

$$n(3) = 200$$

$\therefore n(4) \Rightarrow$  multiple of 4

100, 104, 108, ..., 700

$$T_n = 700 = 100 + (n-1)(4)$$

$$n = 151$$

$$n(4) = 151$$

$n(3 \cap 4) \Rightarrow$  multiple of 3 & 4 both

108, 120, 132, ..., 696

$$T_n = 696 = 108 + (n-1)(12)$$

$$n = 50$$

$$n(3 \cap 4) = 50$$

$$n(3 \cup 4) = n(3) + n(4) - n(3 \cap 4)$$

$$= 200 + 151 - 50$$

$$= 301$$

$n(\overline{3 \cup 4}) = \text{Total} - n(3 \cup 4) =$  neither a multiple of 3 nor a multiple of 4

$$= 601 - 301 = 300$$

7. Let C be the circle of minimum area touching the parabola  $y = 6 - x^2$  and the lines  $y = \sqrt{3}|x|$ . Then, which one of the following points lies on the circle C?

(1) (2, 4)

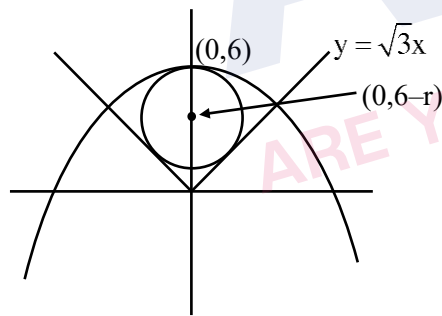
(2) (1, 2)

(3) (2, 2)

(4) (1, 1)

Ans. (1)

Sol.



Equation of circle

$$x^2 + (y - (6 - r))^2 = r^2$$

touches  $\sqrt{3}x - y = 0$

$$p = r$$

$$\frac{|0 - (6 - r)|}{2} = r$$

$$|r - 6| = 2r$$

$$r = 2$$

$$\therefore \text{Circle } x^2 + (y - 4)^2 = 4$$

(2, 4) Satisfies this equation

8. For  $\alpha, \beta \in \mathbb{R}$  and a natural number  $n$ , let

$$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}. \text{ Then } 2A_{10} - A_8 \text{ is}$$

(1)  $4\alpha + 2\beta$

(2)  $2\alpha + 4\beta$

(3)  $2n$

(4) 0

Ans. (1)

Sol.  $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$

$$2A_{10} - A_8 = \begin{vmatrix} 20 & 1 & \frac{n^2}{2} + \alpha \\ 40 & 2 & n^2 - \beta \\ 56 & 3 & \frac{n(3n-1)}{2} \end{vmatrix} - \begin{vmatrix} 8 & 1 & \frac{n^2}{2} + \alpha \\ 16 & 2 & n^2 - \beta \\ 22 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 12 & 1 & \frac{n^2}{2} + \alpha \\ 24 & 2 & n^2 - \beta \\ 34 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 1 & \frac{n^2}{2} + \alpha \\ 0 & 2 & n^2 - \beta \\ -2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$\Rightarrow -2((n^2 - \beta) - (n^2 + 2\alpha))$$

$$\Rightarrow -2(-\beta - 2\alpha) \Rightarrow 4\alpha + 2\beta$$

9. The shortest distance between the lines

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5} \text{ and } \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ is}$$

- (1)  $6\sqrt{3}$  (2)  $4\sqrt{3}$   
(3)  $5\sqrt{3}$  (4)  $8\sqrt{3}$

Ans. (2)

Sol.  $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$  &  $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$

$$S.D = \frac{|(\bar{a}_2 \cdot \bar{a}_1) \cdot (\bar{b}_1 \cdot \bar{b}_2)|}{|\bar{b}_1 \times \bar{b}_2|}$$

$$a_1 = 3, -15, 9 \quad b_1 = 2, -7, 5$$

$$a_2 = -1, 1, 9 \quad b_2 = 2, 1, -3$$

$$a_2 - a_1 = -4, 16, 0$$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix} = \hat{i}(16) - \hat{j}(-16) + \hat{k}(16)$$

$$16(\hat{i} + \hat{j} + \hat{k})$$

$$|\bar{b}_1 \times \bar{b}_2| = 16\sqrt{3}$$

$$\therefore (\bar{a}_2 - \bar{a}_1) \cdot (\bar{b}_1 - \bar{b}_2) = 16[-4 + 16] = (16)(12)$$

$$S.D. = \frac{(16)(12)}{16\sqrt{3}} = 4\sqrt{3}$$

10. A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then 126p is

- (1) 54 (2) 64  
(3) 66 (4) 56

Ans. (1)

Sol.

	A	B
Manufactured	60%	40%
Standard quality	80%	90%

$P(\text{Manufactured at B / found standard quality}) = ?$

A : Found S.Q

B : Manufacture B

C : Manufacture A

$$P(E_1) = \frac{40}{100}$$

$$P(E_2) = \frac{60}{100}$$

$$P(A/E_1) = \frac{90}{100}$$

$$P(A/E_2) = \frac{80}{100}$$

$$\therefore P(E_1/A) = \frac{P(A/E_1) P(E_1)}{P(A/E_1) P(E_1) + P(A/E_2) P(E_2)} = \frac{3}{7}$$

$$\therefore 126 P = 54$$

11. Let,  $\alpha, \beta$  be the distinct roots of the equation

$$x^2 - (t^2 - 5t + 6)x + 1 = 0, t \in \mathbb{R} \text{ and } a_n = \alpha^n + \beta^n.$$

Then the minimum value of  $\frac{a_{2023} + a_{2025}}{a_{2024}}$  is

- (1)  $1/4$  (2)  $-1/2$   
(3)  $-1/4$  (4)  $1/2$

Ans. (3)

Sol. by newton's theorem

$$a_{n+2} - (t^2 - 5t + 6)a_{n+1} + a_n = 0$$

$$\therefore a_{2025} + a_{2023} = (t^2 - 5t + 6) a_{2024}$$

$$\therefore \frac{a_{2025} + a_{2023}}{a_{2024}} = t^2 - 5t + 6$$

$$\therefore t^2 - 5t + 6 = \left(t - \frac{5}{2}\right)^2 - \frac{1}{4}$$

$$\therefore \text{minimum value} = -\frac{1}{4}$$

12. Let the relations  $R_1$  and  $R_2$  on the set  $X = \{1, 2, 3, \dots, 20\}$  be given by  $R_1 = \{(x, y) : 2x - 3y = 2\}$  and  $R_2 = \{(x, y) : -5x + 4y = 0\}$ . If  $M$  and  $N$  be the minimum number of elements required to be added in  $R_1$  and  $R_2$ , respectively, in order to make the relations symmetric, then  $M + N$  equals

- (1) 8 (2) 16  
(3) 12 (4) 10

**Ans. (4)**

**Sol.**  $x = \{1, 2, 3, \dots, 20\}$

$$R_1 = \{(x, y) : 2x - 3y = 2\}$$

$$R_2 = \{(x, y) : -5x + 4y = 0\}$$

$$R_1 = \{(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)\}$$

$$R_2 = \{(4, 5), (8, 10), (12, 15), (16, 20)\}$$

in  $R_1$  6 element needed

in  $R_2$  4 element needed

So, total  $6+4 = 10$  element

13. Let a variable line of slope  $m > 0$  passing through the point  $(4, -9)$  intersect the coordinate axes at the points  $A$  and  $B$ . the minimum value of the sum of the distances of  $A$  and  $B$  from the origin is

- (1) 25 (2) 30  
(3) 15 (4) 10

**Ans. (1)**

**Sol.** equation of line is

$$y + 9 = m(x - 4)$$

$$\therefore A = \left( \frac{9+4m}{m}, 0 \right)$$

$$B = (0, -9 - 4m)$$

$$\therefore OA + OB = \frac{9+4m}{m} + 9 + 4m$$

$$\therefore m > 0$$

$$= 13 + \frac{9}{m} + 4m$$

$$\therefore \frac{4m + \frac{9}{m}}{2} \geq \sqrt{36} \Rightarrow 4m + \frac{9}{m} \geq 12$$

$$\therefore OA + OB \geq 25$$

14. The interval in which the function  $f(x) = x^x$ ,  $x > 0$ , is strictly increasing is

- (1)  $\left(0, \frac{1}{e}\right]$  (2)  $\left[\frac{1}{e^2}, 1\right)$   
(3)  $(0, \infty)$  (4)  $\left[\frac{1}{e}, \infty\right)$

**Ans. (4)**

**Sol.**  $f(x) = x^x$ ;  $x > 0$

$$\ell n y = x \ell n x$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{x}{x} + \ell n x$$

$$\frac{dy}{dx} = x^x (1 + \ell n x)$$

for strictly increasing

$$\frac{dy}{dx} \geq 0 \Rightarrow x^x (1 + \ell n x) \geq 0$$

$$\Rightarrow \ell n x \geq -1$$

$$x \geq e^{-1}$$

$$x \geq \frac{1}{e}$$

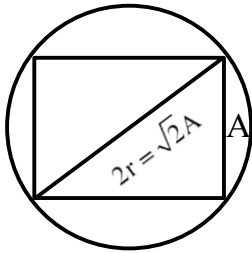
$$x \in \left[ \frac{1}{e}, \infty \right)$$

15. A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are  $m$  and  $n$ , respectively, then  $m + n^2$  is equal to

- (1) 396 (2) 408  
(3) 312 (4) 414

**Ans. (2)**

**Sol.**  $\therefore r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}a^2}{4}}{\frac{3a}{2}} = \frac{a}{2\sqrt{3}} = \frac{12}{2\sqrt{3}} = 2\sqrt{3}$



$\therefore A = r\sqrt{2} = 2\sqrt{6}$

Area =  $m = A^2 = 24$

Perimeter =  $n = 4A = 8\sqrt{6}$

$\therefore m + n^2 = 24 + 384 = 408$

16. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is

- (1) 24 (2) 56  
(3) 16 (4) 48

**Ans. (3)**

**Sol.**  $\therefore$  no. of triangles having no side common with a  $n$

sided polygon =  $\frac{{}^nC_1 \cdot {}^{n-4}C_2}{3}$

$= \frac{{}^8C_1 \cdot {}^4C_2}{3} = 16$

17. Let  $y = y(x)$  be the solution of the differential equation  $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ ,  $y(1) = 0$ . Then  $y(0)$  is

- (1)  $\frac{1}{4}(e^{\pi/2} - 1)$  (2)  $\frac{1}{2}(1 - e^{\pi/2})$   
(3)  $\frac{1}{4}(1 - e^{\pi/2})$  (4)  $\frac{1}{2}(e^{\pi/2} - 1)$

**Ans. (2)**

**Sol.**  $\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$

I.F. =  $e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1}x}$

$y \cdot e^{\tan^{-1}x} = \int \left( \frac{e^{\tan^{-1}x}}{1+x^2} \right) e^{\tan^{-1}x} \cdot dx$

Let  $\tan^{-1}x = z \quad \therefore \frac{dx}{1+x^2} = dz$

$\therefore y \cdot e^z = \int e^{2z} dz = \frac{e^{2z}}{2} + C$

$y \cdot e^{\tan^{-1}x} = \frac{e^{2 \tan^{-1}x}}{2} + C$

$\Rightarrow y = \frac{e^{\tan^{-1}x}}{2} + \frac{C}{e^{\tan^{-1}x}}$

$\therefore y(1) = 0 \Rightarrow 0 = \frac{e^{\pi/4}}{2} + \frac{C}{e^{\pi/4}} \Rightarrow C = \frac{-e^{\pi/2}}{2}$

$\therefore y = \frac{e^{\tan^{-1}x}}{2} - \frac{e^{\pi/2}}{2e^{\tan^{-1}x}}$

$\therefore y(0) = \frac{1 - e^{\pi/2}}{2}$

18. Let  $y = y(x)$  be the solution of the differential equation  $(2x \log_e x) \frac{dy}{dx} + 2y = \frac{3}{x} \log_e x$ ,  $x > 0$  and  $y(e^{-1}) = 0$ . Then,  $y(e)$  is equal to

- (1)  $-\frac{3}{2e}$  (2)  $-\frac{2}{3e}$   
(3)  $-\frac{3}{e}$  (4)  $-\frac{2}{e}$

Ans. (3)

**Sol.**  $\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{3}{2x^2}$

$\therefore$  I.F. =  $e^{\int \frac{1}{x \ln x} dx} = e^{\ln(\ln(x))} = \ln x$

$\therefore y \ln x = \int \frac{3 \ln x}{2x^2} dx$

$= \frac{3 \ln x}{2} \int x^{-2} dx - \int \left( \frac{3}{2x} \cdot \int x^{-2} dx \right) dx$

$= \frac{3 \ln x}{2} \left( -\frac{1}{x} \right) - \int \frac{3}{2x} \left( -\frac{1}{x} \right) dx$

$y \cdot \ln x = \frac{-3 \ln x}{2x} - \frac{3}{2x} + C$

$\therefore y(e^{-1}) = 0$

$\therefore 0(-1) = \frac{3e}{2} - \frac{3e}{2} + C \Rightarrow C = 0$

$\therefore y = \frac{-3 \ln x}{2x} - \frac{3}{2x}$

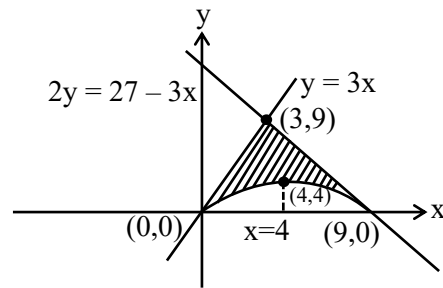
$\therefore y(e) = \frac{-3}{2e} - \frac{3}{2e} = \frac{-3}{e}$

19. Let the area of the region enclosed by the curves  $y = 3x$ ,  $2y = 27 - 3x$  and  $y = 3x - x\sqrt{x}$  be A. Then 10 A is equal to

- (1) 184 (2) 154  
(3) 172 (4) 162

Ans. (4)

**Sol.**  $y = 3x$ ,  $2y = 27 - 3x$  &  $y = 3x - x\sqrt{x}$



$$A = \int_0^3 3x - (3x - x\sqrt{x}) dx + \int_3^9 \left( \frac{27-3x}{2} - (3x - x\sqrt{x}) \right) dx$$

$$A = \int_0^3 x^{3/2} dx + \int_3^9 \frac{27}{2} - \frac{9x}{2} + x^{3/2} dx$$

$$A = \left[ \frac{2x^{5/2}}{5} \right]_0^3 + \frac{27}{2} [x]_3^9 - \frac{9}{2} \left[ \frac{x^2}{2} \right]_3^9 + \left[ \frac{2x^{5/2}}{5} \right]_3^9$$

$$A = \frac{2}{5} (3^{5/2}) + \frac{27}{2} (6) - \frac{9}{4} (72) + \frac{2}{5} (9^{5/2} - 3^{5/2})$$

$$A = \frac{2}{5} (3^{5/2}) + 81 - 162 + \frac{2}{5} \times 3^5 - \frac{2}{5} \times 3^{5/2}$$

$$A = \frac{486}{5} - 81 = \frac{81}{5}$$

$$10A = 162$$

Ans. = 4

20. Let  $f: (-\infty, \infty) - \{0\} \rightarrow \mathbb{R}$  be a differentiable function such that  $f'(1) = \lim_{a \rightarrow \infty} a^2 f\left(\frac{1}{a}\right)$ .

Then  $\lim_{a \rightarrow \infty} \frac{a(a+1)}{2} \tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2 \log_e a$  is equal to

- (1)  $\frac{3}{2} + \frac{\pi}{4}$  (2)  $\frac{3}{8} + \frac{\pi}{4}$   
(3)  $\frac{5}{2} + \frac{\pi}{8}$  (4)  $\frac{3}{4} + \frac{\pi}{8}$

Ans. (3)

**Sol.**  $f: (-\infty, \infty) - \{0\} \rightarrow \mathbb{R}$

$$f'(1) = \lim_{a \rightarrow \infty} a^2 f\left(\frac{1}{a}\right)$$

$$\lim_{a \rightarrow \infty} \frac{a(a+1)}{2} \tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\ln(a)$$

$$\lim_{a \rightarrow \infty} a^2 \left( \frac{\left(1 + \frac{1}{a}\right)}{2} \tan^{-1}\left(\frac{1}{a}\right) + 1 - \frac{2}{a^2} \ln(a) \right)$$

$$f(x) = \frac{1}{2} (1+x) \tan^{-1}(x) + 1 - 2x^2 \ln(x)$$

$$f'(x) = \frac{1}{2} \left( \frac{1+x}{1+x^2} + \tan^{-1}(x) + 4x \ln(x) \right) + 2x$$

$$f'(1) = \frac{1}{2} \left( 1 + \frac{\pi}{4} \right) + 2$$

$$f'(1) = \frac{5}{2} + \frac{\pi}{8}$$

Ans. (3)

### SECTION-B

**21.** Let  $\alpha\beta\gamma = 45$ ;  $\alpha, \beta, \gamma \in \mathbb{R}$ . If  $x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0)$  for some  $x, y, z \in \mathbb{R}$ ,  $xyz \neq 0$ , then  $6\alpha + 4\beta + \gamma$  is equal to \_\_\_\_\_

**Ans. (55)**

**Sol.**  $\alpha\beta\gamma = 45, \alpha\beta\gamma \in \mathbb{R}$

$$x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0)$$

$$x, y, z \in \mathbb{R}, xyz \neq 0$$

$$\alpha x + y + 2z = 0$$

$$x + \beta y + 3z = 0$$

$$2x + 2y + \gamma z = 0$$

$$xyz \neq 0 \Rightarrow \text{non-trivial}$$

$$\begin{vmatrix} \alpha & 1 & 2 \\ 1 & \beta & 3 \\ 2 & 2 & \gamma \end{vmatrix} = 0$$

$$\Rightarrow \alpha(\beta\gamma - 6) - 1(\gamma - 6) + 2(2 - 2\beta) = 0$$

$$\Rightarrow \alpha\beta\gamma - 6\alpha - \gamma + 6 + 4 - 4\beta = 0$$

$$\Rightarrow 6\alpha + 4\beta + \gamma = 55$$

**22.** Let a conic C pass through the point  $(4, -2)$  and  $P(x, y)$ ,  $x \geq 3$ , be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and  $(3, -5)$ . If the focal distance of the point  $(7, 1)$  on C is d, then  $12d$  equals \_\_\_\_\_.

**Ans. (75)**

**Sol.**  $P(x, y)$  &  $x \geq 3$

$$\text{Slope of line at } P(x, y) \text{ will be } \frac{dy}{dx} = \frac{1}{2} \left( \frac{y+5}{x-3} \right)$$

$$\Rightarrow 2 \frac{dy}{(y+5)} = \frac{1}{(x-3)} dx$$

$$\Rightarrow 2\ln(y+5) = \ln(x-3) + C$$

Passes through  $(4, -2)$

$$\Rightarrow 2\ln(3) = \ln(1) + C$$

$$\Rightarrow C = 2\ln(3)$$

$$\Rightarrow 2\ln(y+5) = \ln(x-3) + 2\ln(3)$$

$$\Rightarrow 2 \left( \ln \left( \frac{y+5}{3} \right) \right) = \ln(x-3)$$

$$\Rightarrow \left( \frac{y+5}{3} \right)^2 = (x-3)$$

$$\Rightarrow (y+5)^2 = 9(x-3)$$

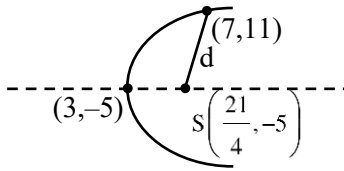
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Parabola

$$4a = 9$$

$$a = \frac{9}{4}$$





$$d = \sqrt{\left(\frac{7}{4}\right)^2 + 6^2}$$

$$d = \frac{\sqrt{625}}{4}$$

$$d = \frac{25}{4}$$

$$12d = 75$$

23. Let  $r_k = \frac{\int_0^1 (1-x^7)^k dx}{\int_0^1 (1-x^7)^{k+1} dx}$ ,  $k \in \mathbb{N}$ . Then the value of

$$\sum_{k=1}^{10} \frac{1}{7(r_k - 1)}$$
 is equal to \_\_\_\_\_.

**Ans. (65)**

**Sol.**  $I_K = \int_0^1 (1-x^7)^K dx$

$$I_K = (1-x^7)^K x \Big|_0^1 + 7K \int_0^1 (1-x^7)^{K-1} x^6 dx$$

$$I_K = -7K \int_0^1 (1-x^7)^{K-1} ((1-x^7) - 1) dx$$

$$I_K = -7K I_K + 7K I_{K-1}$$

$$\Rightarrow \frac{I_K}{I_{K+1}} = \frac{7K+8}{7K+7}$$

$$r_K = \frac{7K+8}{7K+7}$$

$$r_K - 1 = \frac{1}{7(K+1)}$$

$$\Rightarrow 7(r_K - 1) = \frac{1}{K+1}$$

$$\sum_{K=1}^{10} (K+1) = 11(6) - 1 = 65$$

24. Let  $x_1, x_2, x_3, x_4$  be the solution of the equation

$$4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0 \text{ and}$$

$$(4+x_1^2)(4+x_2^2)(4+x_3^2)(4+x_4^2) = \frac{125}{16} m.$$

Then the value of  $m$  is \_\_\_\_\_.

**Ans. (221)**

**Sol.**  $4x^4 + 8x^3 - 17x^2 - 12x + 9$

$$= 4(x-x_1)(x-x_2)(x-x_3)(x-x_4)$$

Put  $x = 2i$  &  $-2i$

$$64 - 64i + 68 - 24i + 9 = (2i-x_1)(2i-x_2)(2i-x_3)(2i-x_4)$$

$$= 141 - 88i \quad \dots\dots(1)$$

$$64 + 64i + 68 + 24i + 9 = 4(-2i-x_1)(-2i-x_2)(-2i-x_3)(-2i-x_4)$$

$$= 141 + 88i \quad \dots\dots(2)$$

$$\frac{125}{16} m = \frac{141^2 + 88^2}{16}$$

$$m = 221$$

25. Let  $L_1, L_2$  be the lines passing through the point  $P(0, 1)$  and touching the parabola

$$9x^2 + 12x + 18y - 14 = 0. \text{ Let } Q \text{ and } R \text{ be the}$$

points on the lines  $L_1$  and  $L_2$  such that the  $\Delta PQR$

is an isosceles triangle with base  $QR$ . If the slopes

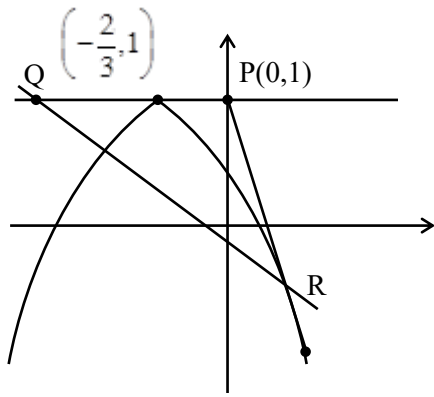
of the lines  $QR$  are  $m_1$  and  $m_2$ . then  $16(m_1^2 + m_2^2)$  is equal to \_\_\_\_\_.

**Ans. (68)**

**Sol.**  $9x^2 + 12x + 4 = -18(y-1)$

$$(3x+2)^2 = -18(y-1)$$

$$\left(x + \frac{2}{3}\right)^2 = -2(y-1)$$



(0, 1)

$$y = mx + 1$$

$$\left(x + \frac{2}{3}\right)^2 = -2(y - 1)$$

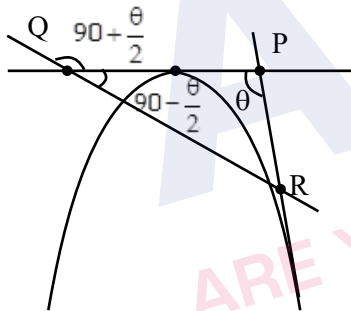
$$(3x + 2)^2 = -18mx$$

$$9x^2 + (12 + 18m)x + 4 = 0$$

$$4(6 + 9m)^2 = 4(36)$$

$$6 + 9m = 6, -6$$

$$m = 0, \frac{-4}{3}$$



$$\tan \theta = -\frac{4}{3}$$

$$\frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} = \frac{-4}{3}$$

$$\left(\tan \frac{\theta}{2} - 2\right)\left(2 \tan \frac{\theta}{2} + 1\right) = 0$$

$$\tan \frac{\theta}{2} = 2, \frac{-1}{2}$$

$$m_{QR} = \tan\left(90 + \frac{\theta}{2}\right)$$

$$= -\cot \frac{\theta}{2}$$

$$m_1 = \frac{-1}{2}$$

$$m_2 = \frac{-1}{-1/2} = 2$$

$$16(m_1^2 + m_2^2) = 16\left(\frac{1}{4} + 4\right)$$

$$= 4 + 64 = 68$$

26. If the second, third and fourth terms in the expansion of  $(x + y)^n$  are 135, 30 and  $\frac{10}{3}$ , respectively, then  $6(n^3 + x^2 + y)$  is equal to \_\_\_\_\_.

**Ans. (806)**

**Sol.**  ${}^nC_1 x^{n-1} y = 135$  ....(i)

$${}^nC_2 x^{n-2} y^2 = 30$$
 ....(ii)

$${}^nC_3 x^{n-3} y^3 = \frac{10}{3}$$
 ....(iii)

By (i)  
(ii)

$$\frac{{}^nC_1 x}{{}^nC_2 y} = \frac{9}{2}$$
 ....(iv)

By (ii)  
(iii)

$$\frac{{}^nC_2 x}{{}^nC_3 y} = 9$$
 ....(v)

By (iv)  
(v)

$$\frac{{}^nC_1 {}^nC_3}{{}^nC_2 {}^nC_2} = \frac{1}{2}$$

$$\frac{2n^2(n-1)(n-2)}{6} = \frac{n(n-1)}{2} \frac{n(n-1)}{2}$$

$$4n - 8 = 3n - 3$$

$$\Rightarrow \boxed{n = 5}$$

put in (v)

$$\frac{x}{y} = 9$$

$$x = 9y$$

put in (i)

$${}^5C_1 x^4 \left( \frac{x}{9} \right) = 135$$

$$x^5 = 27 \times 9$$

$$\Rightarrow x = 3, \quad y = \frac{1}{3}$$

$$6(n^3 + x^2 + y)$$

$$= 6 \left( 125 + 9 + \frac{1}{3} \right)$$

$$= 806$$

27. Let the first term of a series be  $T_1 = 6$  and its  $r^{\text{th}}$  term  $T_r = 3 T_{r-1} + 6^r$ ,  $r = 2, 3, \dots, n$ . If the sum of the first  $n$  terms of this series is  $\frac{1}{5}(n^2 - 12n + 39)$   $(4.6^n - 5.3^n + 1)$ . Then  $n$  is equal to \_\_\_\_\_.

**Ans. (6)**

**Sol.**  $T_r = 3T_{r-1} + 6^r$ ,  $r = 2, 3, 4, \dots, n$

$$T_2 = 3.T_1 + 6^2$$

$$T_2 = 3.6 + 6^2 \quad \dots(1)$$

$$T_3 = 3T_2 + 6^3$$

$$T_3 = 3T_2 + 6^3$$

$$T_3 = 3(3.6 + 6^2) + 6^3$$

$$T_3 = 3^2.6 + 3.6^2 + 6^3 \quad \dots(2)$$

$$T_r = 3^{r-1}.6 + 3^{r-2}.6^2 + \dots + 6^r$$

$$T_r = 3^{r-1} \cdot 6 \left[ 1 + \frac{6}{3} + \left( \frac{6}{3} \right)^2 + \dots + \left( \frac{6}{3} \right)^{r-1} \right]$$

$$T_r = 3^{r-1} \cdot 6(1 + 2 + 2^2 + \dots + 2^{r-1})$$

$$T_r = 6 \cdot 3^{r-1} \cdot \frac{(1-2^r)}{(-1)}$$

$$T_r = 6.3^{r-1} \cdot (2^r - 1)$$

$$T_r = \frac{6 \cdot 3^r}{3} \cdot (2^r - 1)$$

$$T_r = 2 \cdot (6^r - 3^r)$$

$$S_n = 2 \sum (6^r - 3^r)$$

$$S_n = 2 \cdot \left[ \frac{6(6^n - 1)}{5} - \frac{3(3^n - 1)}{2} \right]$$

$$S_n = 2 \left[ \frac{12(6^n - 1) - 15(3^n - 1)}{10} \right]$$

$$S_n = \frac{3}{5} [4.6^4 - 5.3^n + 1]$$

$$\therefore n^2 - 12n + 39 = 3$$

$$n^2 - 12n + 36 = 0$$

$$n = 6$$

28. For  $n \in \mathbb{N}$ , if  $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1}n = \frac{\pi}{4}$ ,

then  $n$  is equal to \_\_\_\_\_.

**Ans. (47)**

**Sol.**  $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1}n = \frac{\pi}{4}$

$$\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{n} = \frac{\pi}{4}$$

$$\tan^{-1} \left( \frac{46}{48} \right) + \tan^{-1} \frac{1}{n} = \frac{\pi}{4}$$

$$\tan^{-1} \left( \frac{23}{24} \right) + \tan^{-1} \frac{1}{n} = \frac{\pi}{4}$$

$$\tan^{-1} \frac{1}{n} = \tan^{-1} 1 - \tan^{-1} \frac{23}{24}$$

$$\tan^{-1} \frac{1}{n} = \tan^{-1} \left( \frac{1 - \frac{23}{24}}{1 + \frac{23}{24}} \right)$$

$$\tan^{-1} \frac{1}{n} = \tan^{-1} \left( \frac{1}{\frac{24}{47}} \right)$$

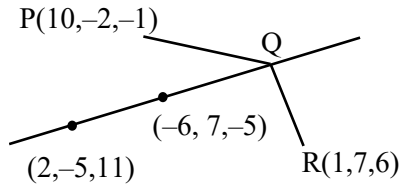
$$\tan^{-1} \frac{1}{n} = \tan^{-1} \frac{1}{47}$$

$$n = 47$$

29. Let P be the point (10, -2, -1) and Q be the foot of the perpendicular drawn from the point R(1, 7, 6) on the line passing through the points (2, -5, 11) and (-6, 7, -5). Then the length of the line segment PQ is equal to \_\_\_\_\_.

**Ans. (13)**

**Sol.**



$$\text{Line : } \frac{x+6}{-8} = \frac{y-7}{12} = \frac{z+5}{-16}$$

$$\frac{x+6}{-8} = \frac{y-7}{12} = \frac{z+5}{-16} = \lambda$$

$$Q(2\lambda - 6, 7 - 3\lambda, 4\lambda - 5)$$

$$\overrightarrow{QR}(2\lambda - 7, -3\lambda, 4\lambda - 11)$$

$$\overrightarrow{QR} \cdot \text{dr's of line} = 0$$

$$4\lambda - 14 + 9\lambda + 16\lambda - 44 = 0$$

$$29\lambda = 58 \Rightarrow \lambda = 2$$

$$Q(-2, 1, 3)$$

$$PQ = \sqrt{144 + 9 + 16} = \sqrt{169} = 13$$

30. Let  $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ ,  $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ , and a vector  $\vec{c}$  be such that  $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 13\hat{k}$ . If  $\vec{a} \cdot \vec{c} = 13$ , then  $(24 - \vec{b} \cdot \vec{c})$  is equal to \_\_\_\_\_.

**Ans. (46)**

**Sol.**  $\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} = (1, 8, 13)$

$$\vec{a} \times (\vec{a} \times \vec{b}) + \vec{a} \times (\vec{a} \times \vec{c}) + \vec{a} \times (\vec{b} \times \vec{c})$$

$$= \vec{a} \times (\hat{i} + 8\hat{j} + 13\hat{k})$$

$$(\vec{a} \cdot \vec{b})\vec{a} - a^2\vec{b} + (\vec{a} \cdot \vec{c})\vec{a} - a^2\vec{c} + (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \vec{a} \times (\hat{i} + 8\hat{j} + 13\hat{k})$$

$$\Rightarrow -26\vec{a} - 29\vec{b} + 13\vec{a} - 29\vec{c} + 13\vec{b} + 26\vec{c} = \vec{a} \times (\hat{i} + 8\hat{j} + 13\hat{k})$$

$$\Rightarrow -13\vec{a} - 16\vec{b} - 3\vec{c} = \vec{a} \times (\hat{i} + 8\hat{j} + 13\hat{k})$$

$$\Rightarrow -13\vec{a} \cdot \vec{b} - 16b^2 - 3\vec{b} \cdot \vec{c} = \left\{ \vec{a} \times (\hat{i} + 8\hat{j} + 13\hat{k}) \right\} \cdot \vec{b}$$

$$\Rightarrow (-13)(-26) - 16(50) - 3\vec{b} \cdot \vec{c} = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 8 & 13 \\ 3 & 4 & -5 \end{vmatrix}$$

$$\Rightarrow -462 - 3\vec{b} \cdot \vec{c} = -396$$

$$\Rightarrow \vec{b} \cdot \vec{c} = -22$$

$$\text{Hence } 24 - \vec{b} \cdot \vec{c} = 46$$

## PHYSICS

### SECTION-A

31. To find the spring constant ( $k$ ) of a spring experimentally, a student commits 2% positive error in the measurement of time and 1% negative error in measurement of mass. The percentage error in determining value of  $k$  is :

- (1) 3% (2) 1%  
(3) 4% (4) 5%

Ans. (4)

Sol.  $T = 2\pi\sqrt{\frac{m}{k}}$

$$T^2 \propto \frac{m}{k}$$

$$\frac{2\Delta T}{T}\% = \frac{\Delta m}{m}\% - \frac{\Delta k}{k}\%$$

$$\frac{\Delta k}{k}\% = \frac{\Delta m}{m}\% - \frac{2\Delta T}{T}\%$$

$$\frac{\Delta k}{k}\% = (-1)\% - 2(2)\% = -5\% = 5\%$$

32. A bullet of mass 50 g is fired with a speed 100 m/s on a plywood and emerges with 40 m/s. The percentage loss of kinetic energy is :

- (1) 32% (2) 44%  
(3) 16% (4) 84%

Ans. (4)

Sol.  $K_i = \frac{1}{2}m(100)^2$

$$K_f = \frac{1}{2}m(40)^2$$

$$\% \text{loss} = \frac{|K_f - K_i|}{K_i} \times 100$$

$$= \frac{\left| \frac{1}{2}m(40)^2 - \frac{1}{2}m(100)^2 \right|}{\frac{1}{2}m(100)^2} \times 100$$

$$= \frac{|1600 - 100 \times 100|}{100} = 84\%$$

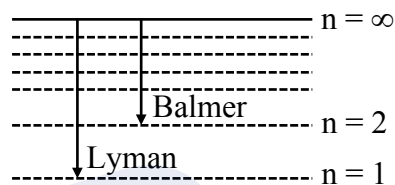
## TEST PAPER WITH SOLUTION

33. The ratio of the shortest wavelength of Balmer series to the shortest wavelength of Lyman series for hydrogen atom is :

- (1) 4 : 1 (2) 1 : 2  
(3) 1 : 4 (4) 2 : 1

Ans. (1)

Sol



$$\frac{1}{\lambda} = RZ^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\frac{1}{\lambda_L} = RZ^2 \left( \frac{1}{1^2} \right)$$

$$\frac{1}{\lambda_B} = RZ^2 \left( \frac{1}{2^2} \right)$$

$$\frac{\lambda_B}{\lambda_L} = 4 : 1$$

34. To project a body of mass  $m$  from earth's surface to infinity, the required kinetic energy is (assume, the radius of earth is  $R_E$ ,  $g$  = acceleration due to gravity on the surface of earth) :

- (1)  $2mgR_E$  (2)  $mgR_E$   
(3)  $\frac{1}{2}mgR_E$  (4)  $4mgR_E$

Ans. (2)

Sol.  $\frac{1}{2}mv_e^2 = \frac{GMm}{R_E}$

$$g = \frac{GM}{R_E^2}$$

$$K = mgR_E$$

35. Electromagnetic waves travel in a medium with speed of  $1.5 \times 10^8 \text{ ms}^{-1}$ . The relative permeability of the medium is 2.0. The relative permittivity will be :

- (1) 5 (2) 1  
(3) 4 (4) 2

Ans. (4)

Sol. 
$$\frac{\epsilon_m \times \mu_m}{\epsilon_0 \times \mu_0} = \frac{1}{\frac{v^2}{c^2}}$$

$$\epsilon_r \times \mu_r = \frac{c^2}{v^2}$$

$$\epsilon_r \times 2 = \frac{(3 \times 10^8)^2}{(1.5 \times 10^8)^2}$$

$$\epsilon_r \times 2 = 4$$

$$\epsilon_r = 2$$

36. Which of the following phenomena does not explain by wave nature of light.

- (A) reflection (B) diffraction  
(C) photoelectric effect (D) interference  
(E) polarization

Choose the **most appropriate** answer from the options given below :

- (1) E only (2) C only  
(3) B, D only (4) A, C only

Ans. (2)

Sol. (Theory)

Photoelectric effect prove particle nature of light.

37. While measuring diameter of wire using screw gauge the following readings were noted. Main scale reading is 1 mm and circular scale reading is equal to 42 divisions. Pitch of screw gauge is 1 mm and it has 100 divisions on circular scale. The

diameter of the wire is  $\frac{x}{50}$  mm. The value of x is :

- (1) 142 (2) 71  
(3) 42 (4) 21

Ans. (2)

- Sol. MSR = 1mm, CSR = 42, pitch = 1 mm

$$LC = \frac{\text{pitch}}{\text{No. of CSD}} = \left( \frac{1}{100} \right) = 0.01 \text{ mm}$$

$$\text{Diameter} = \text{MSR} + LC \times \text{CSD}$$

$$\text{Diameter} = 1 + (0.01) \times 42 \text{ mm}$$

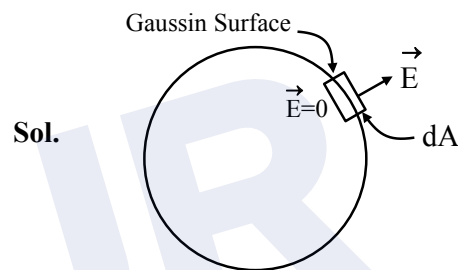
$$\text{Diameter} = 1.42 \text{ mm} = \frac{x}{50}$$

$$\therefore x = 71$$

38.  $\sigma$  is the uniform surface charge density of a thin spherical shell of radius R. The electric field at any point on the surface of the spherical shell is :

- (1)  $\sigma/\epsilon_0 R$  (2)  $\sigma/2\epsilon_0$   
(3)  $\sigma/\epsilon_0$  (4)  $\sigma/4\epsilon_0$

Ans. (3)



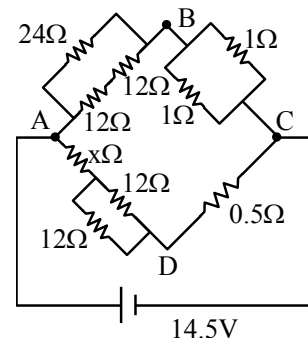
Sol.

$$\text{By Gauss law } \int \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$EdA = \frac{\sigma \times dA}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

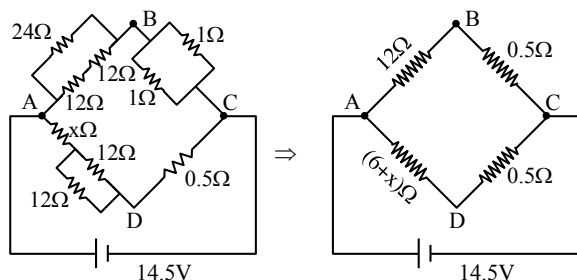
39. The value of unknown resistance (x) for which the potential difference between B and D will be zero in the arrangement shown, is :



- (1)  $3 \Omega$  (2)  $9 \Omega$   
(3)  $6 \Omega$  (4)  $42 \Omega$

Ans. (3)

**Sol.**



In case of balanced Wheatstone Bridge

$$\frac{V_{AB}}{V_{AD}} = \frac{V_{BC}}{V_{CD}} \Rightarrow \frac{12}{6+x} = \frac{0.5}{0.5}$$

$$x = 6 \Omega$$

40. The specific heat at constant pressure of a real gas obeying  $PV^2 = RT$  equation is :

- (1)  $C_V + R$  (2)  $\frac{R}{3} + C_V$   
(3)  $R$  (4)  $C_V + \frac{R}{2V}$

**Ans. (4)**

**Sol.**  $dQ = du + dW$

$$CdT = C_V dT + PdV \quad \dots (1)$$

$$\therefore PV^2 = RT$$

$$P = \text{constant}$$

$$P(2VdV) = RdT$$

$$PdV = \frac{RdT}{2V}$$

Put in equation (1)

$$C = C_V + \frac{R}{2V}$$

41. Match List I with List II

	LIST I		LIST II
A.	Torque	I.	$[M^1 L^1 T^{-2} A^{-2}]$
B.	Magnetic field	II.	$[L^2 A^1]$
C.	Magnetic moment	III.	$[M^1 T^{-2} A^{-1}]$
D.	Permeability of free space	IV.	$[M^1 L^2 T^{-2}]$

Choose the **correct** answer from the options given below :

- (1) A-I, B-III, C-II, D-IV  
(2) A-IV, B-III, C-II, D-I  
(3) A-III, B-I, C-II, D-IV  
(4) A-IV, B-II, C-III, D-I

**Ans. (2)**

**Sol.**  $[\vec{\tau}] = [\vec{r} \times \vec{F}] = [ML^2 T^{-2}]$

$$[F] = [qVB]$$

$$\Rightarrow B = \left( \frac{F}{qV} \right) = \left[ \frac{MLT^{-2}}{ATLT^{-1}} \right] = [MA^{-1}T^{-2}]$$

$$[M] = [I \times A] = [AL^2]$$

$$B = \frac{\mu_0 Idl \sin \theta}{4\pi r^2}$$

$$\Rightarrow [\mu] = \left[ \frac{Br^2}{Idl} \right] = \left[ \frac{MT^{-2}A^{-1} \times L^2}{AL} \right] = [MLT^{-2}A^{-2}]$$

42. Given below are two statements :

**Statement I :** In an LCR series circuit, current is maximum at resonance.

**Statement II :** Current in a purely resistive circuit can never be less than that in a series LCR circuit when connected to same voltage source.

In the light of the above statements, choose the **correct** from the options given below :

- (1) Statement I is true but Statement II is false  
(2) Statement I is false but Statement II is true  
(3) Both Statement I and Statement II are true  
(4) Both Statement I and Statement II are false

**Ans. (3)**

**Sol. Statement-I**

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} \text{ at resonance } X_L = X_C$$

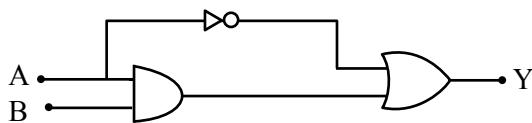
$$\text{Thus, } I_m = \frac{V_m}{R}$$

$\therefore$  Impedence is minimum therefore  $I$  is maximum at resonance.

**Statement-II**

$$I = \left( \frac{V}{R} \right) \text{ in purely resistive circuit.}$$

43. The correct truth table for the following logic circuit is :



Options :

(1)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

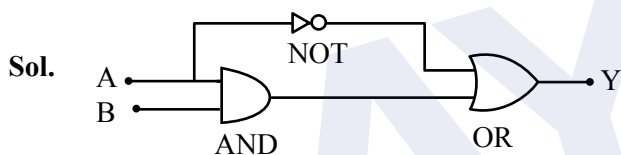
(3)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(4)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Ans. (2)



44. A sample contains mixture of helium and oxygen gas. The ratio of root mean square speed of helium and oxygen in the sample, is :

(1)  $\frac{1}{32}$

(2)  $\frac{2\sqrt{2}}{1}$

(3)  $\frac{1}{4}$

(4)  $\frac{1}{2\sqrt{2}}$

Ans. (2)

Sol.

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M_w}}$$

$$\Rightarrow \frac{V_{\text{O}_2}}{V_{\text{He}}} = \sqrt{\frac{M_{w,\text{He}}}{M_{w,\text{O}_2}}}$$

$$= \sqrt{\frac{4}{32}} = \frac{1}{2\sqrt{2}}$$

$$\frac{V_{\text{He}}}{V_{\text{O}_2}} = \frac{2\sqrt{2}}{1}$$

45. A light string passing over a smooth light pulley connects two blocks of masses  $m_1$  and  $m_2$  (where  $m_2 > m_1$ ). If the acceleration of the system is  $\frac{g}{\sqrt{2}}$ , then the ratio of the masses  $\frac{m_1}{m_2}$  is :

(1)  $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

(2)  $\frac{1+\sqrt{5}}{\sqrt{5}-1}$

(3)  $\frac{1+\sqrt{5}}{\sqrt{2}-1}$

(4)  $\frac{\sqrt{3}+1}{\sqrt{2}-1}$

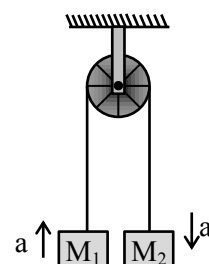
Ans. (1)

Sol.  $a = \left( \frac{M_2 - M_1}{M_1 + M_2} \right) g$

$$\frac{g}{\sqrt{2}} = \left( \frac{M_2 - M_1}{M_1 + M_2} \right) g$$

$$(M_1 + M_2) = \sqrt{2}M_2 - \sqrt{2}M_1$$

$$\frac{M_1}{M_2} = \left( \frac{\sqrt{2}-1}{\sqrt{2}+1} \right)$$



46. Four particles A, B, C, D of mass  $\frac{m}{2}$ ,  $m$ ,  $2m$ ,  $4m$ , have same momentum, respectively. The particle with maximum kinetic energy is :

(1) D

(2) C

(3) A

(4) B

Ans. (3)

Sol.  $KE = \frac{p^2}{2m}$

Same momentum, so less mass means more KE.

So  $\frac{m}{2}$  will have max. KE.

47. A train starting from rest first accelerates uniformly up to a speed of 80 km/h for time  $t$ , then it moves with a constant speed for time  $3t$ . The average speed of the train for this duration of journey will be (in km/h) :

(1) 80

(2) 70

(3) 30

(4) 40

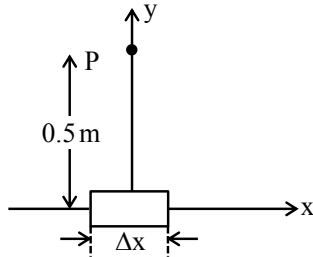
Ans. (2)



**Sol.** Average speed =  $\frac{\text{total distance}}{\text{time taken}}$

$$= \frac{\frac{80 \times t}{2} + 80 \times 3t}{4t} = 70 \text{ km/hr.}$$

48. An element  $\Delta l = \Delta x \hat{i}$  is placed at the origin and carries a large current  $I = 10\text{A}$ . The magnetic field on the y-axis at a distance of 0.5 m from the elements  $\Delta x$  of 1 cm length is :



- (1)  $4 \times 10^{-8} \text{T}$  (2)  $8 \times 10^{-8} \text{T}$   
(3)  $12 \times 10^{-8} \text{T}$  (4)  $10 \times 10^{-8} \text{T}$

**Ans. (1)**

**Sol.**

$$\vec{r} = 0.5 \hat{j}$$

$$\vec{dB} = \frac{\mu_0 I (\vec{dl} \times \vec{r})}{4\pi r^3} \text{ (Tesla)}$$

$$= \frac{10^{-7} \times 10 \times \left(\frac{1}{2} \times \frac{1}{100}\right) (+\hat{k})}{\left(\frac{1}{2}\right)^3} = 4 \times 10^{-8} \text{T} (+\hat{k})$$

49. A small ball of mass  $m$  and density  $\rho$  is dropped in a viscous liquid of density  $\rho_0$ . After sometime, the ball falls with constant velocity. The viscous force on the ball is :

- (1)  $mg \left( \frac{\rho_0}{\rho} - 1 \right)$  (2)  $mg \left( 1 + \frac{\rho}{\rho_0} \right)$   
(3)  $mg(1 - \rho\rho_0)$  (4)  $mg \left( 1 - \frac{\rho_0}{\rho} \right)$

**Ans. (4)**

**Sol.**  $mg - F_B - F_v = ma$   
 $a = 0$  for constant velocity  
 $mg - F_B = F_v$

$$F_v = mg - v \rho_0 g = mg - \frac{m}{\rho} \rho_0 g = mg \left( 1 - \frac{\rho_0}{\rho} \right)$$

50. In photoelectric experiment energy of 2.48 eV irradiates a photo sensitive material. The stopping potential was measured to be 0.5 V. Work function of the photo sensitive material is :  
(1) 0.5 eV (2) 1.68 eV  
(3) 2.48 eV (4) 1.98 eV

**Ans. (4)**

**Sol.**  $eV_s = hv - \phi$   
 $0.5 \text{ V} = 2.48 - \phi$   
work function ( $\phi$ ) =  $2.48 \text{ V} - 0.5 \text{ V} = 1.98 \text{ V}$

### SECTION-B

51. If the radius of earth is reduced to three-fourth of its present value without change in its mass then value of duration of the day of earth will be \_\_\_\_\_ hours 30 minutes.

**Ans. (13)**

**Sol.** By conservation of angular momentum

$$I_1 \omega_1 = I_2 \omega_2$$

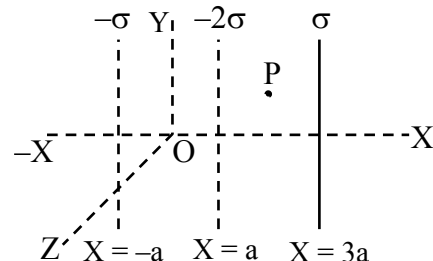
$$\left( \frac{2}{5} MR^2 \right) \frac{2\pi}{T_1} = \frac{2}{5} M \left( \frac{3}{4} R \right)^2 \frac{2\pi}{T_2}$$

$$\frac{1}{T_1} = \frac{9}{16 T_2}$$

$$\frac{1}{T_2} = \frac{9}{16} \times T_1 = \frac{9}{16} \times 24 \text{ hr} = \frac{27}{2} \text{ hr} = 13 \text{ hr } 30 \text{ mins.}$$

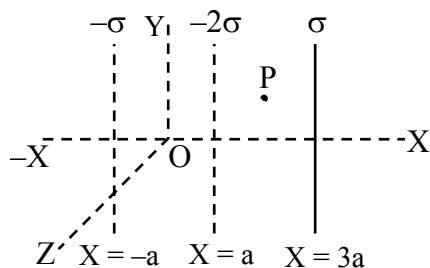
52. Three infinitely long charged thin sheets are placed as shown in figure. The magnitude of electric field at the point P is  $\frac{x\sigma}{\epsilon_0}$ . The value of  $x$  is \_\_\_\_\_

(all quantities are measured in SI units).



**Ans. (2)**

Sol.



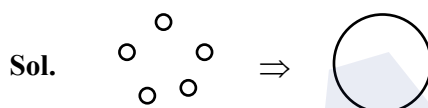
$$\vec{E}_p = \left( \frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} \right) (-\hat{i})$$

$$= -\frac{2\sigma}{\epsilon_0} \hat{i}$$

53. A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of 1000 droplets to that of energy of big drop is  $\frac{10}{x}$ .

The value of x is \_\_\_\_\_.

Ans. (1)



1000 drops      Big drop

$$1000 \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$10r = R$$

$$R = 10r$$

$$\frac{\text{S.E. of 1000 drops}}{\text{S.E. of Big drop}} = \frac{1000(4\pi r^2)T}{4\pi R^2 T}$$

$$= \frac{1000 \times r^2}{(10r)^2} = 10 = \frac{10}{x}$$

$$\therefore x = 1$$

54. When a dc voltage of 100V is applied to an inductor, a dc current of 5A flows through it. When an ac voltage of 200V peak value is connected to inductor, its inductive reactance is found to be  $20\sqrt{3} \Omega$ . The power dissipated in the circuit is \_\_\_\_\_ W.

Ans. (250)

Sol. For DC voltage

$$R = \frac{V}{I} = \frac{100}{5} = 20 \Omega$$

for AC voltage

$$X_L = 20\sqrt{3} \Omega$$

$$R = 20 \Omega$$

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{3 \times 400 + 400} = 40 \Omega$$

$$\text{Power} = i_{\text{rms}}^2 R$$

$$= \left( \frac{V_{\text{rms}}}{Z} \right)^2 \times R = \left( \frac{\frac{200}{\sqrt{2}}}{40} \right)^2 \times 20 = 250 \text{ W}$$

55. The refractive index of prism is  $\mu = \sqrt{3}$  and the ratio of the angle of minimum deviation to the angle of prism is one. The value of angle of prism is \_\_\_\_\_°.

Ans. (60)

Sol. For  $\delta_{\min}$

$$i = e$$

$$r_1 = r_2 = \frac{A}{2}$$

$$\frac{\delta_{\min}}{A} = 1$$

$$\frac{2i - A}{A} = 1$$

$$2i = 2A$$

$$i = A$$

Snell's law

$$1 \times \sin i = \mu \sin r$$

$$\sin i = \mu \sin \left( \frac{A}{2} \right)$$

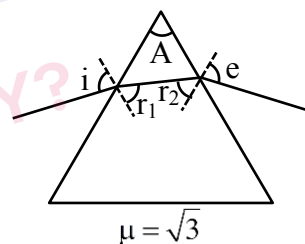
$$\sin A = \mu \sin \left( \frac{A}{2} \right)$$

$$2 \sin \frac{A}{2} \cos \frac{A}{2} = \sqrt{3} \sin \left( \frac{A}{2} \right)$$

$$\cos \left( \frac{A}{2} \right) = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{A}{2} = 30^\circ$$

$$\therefore A = 60^\circ$$



56. A wire of resistance  $R$  and radius  $r$  is stretched till its radius became  $r/2$ . If new resistance of the stretched wire is  $xR$ , then value of  $x$  is \_\_\_\_\_.

Ans. (16)

Sol. We know  $R = \frac{\rho l}{A}$ ,  $R \propto \frac{l}{r^2}$

As we stretch the wire, its length will increase but its radius will decrease keeping the volume constant

$$V_i = V_f$$

$$\pi r^2 l = \pi \frac{r_f^2}{4} l_f$$

$$l_f = 4l$$

$$\frac{R_{\text{new}}}{R_{\text{old}}} = \left( \frac{4l}{\frac{r^2}{4}} \right) \frac{r^2}{l} = 16$$

$$R_{\text{new}} = 16R$$

$$\therefore x = 16$$

57. Radius of a certain orbit of hydrogen atom is  $8.48 \text{ \AA}$ . If energy of electron in this orbit is  $E/x$ , then  $x =$  \_\_\_\_\_.

(Given  $a_0 = 0.529 \text{ \AA}$ ,  $E$  = energy of electron in ground state)

Ans. (16)

Sol. We know

$$r = 0.529 \frac{n^2}{Z} \Rightarrow 8.48 = 0.529 \frac{n^2}{1}$$

$$n^2 = 16 \Rightarrow n = 4$$

We know

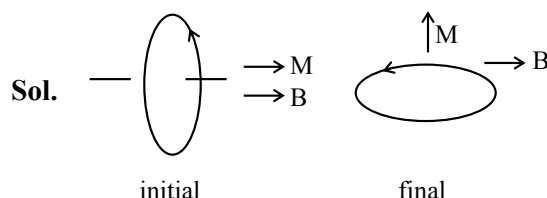
$$E \propto \frac{1}{n^2}$$

$$E_{n^{\text{th}}} = \frac{E}{16}$$

$$x = 16$$

58. A circular coil having 200 turns,  $2.5 \times 10^{-4} \text{ m}^2$  area and carrying  $100 \mu\text{A}$  current is placed in a uniform magnetic field of  $1 \text{ T}$ . Initially the magnetic dipole moment ( $\vec{M}$ ) was directed along  $\vec{B}$ . Amount of work, required to rotate the coil through  $90^\circ$  from its initial orientation such that  $\vec{M}$  becomes perpendicular to  $\vec{B}$ , is \_\_\_\_\_  $\mu\text{J}$ .

Ans. (5)



Sol.

We know

$$\begin{aligned} W_{\text{ext}} &= \Delta U + \Delta KE \quad (\text{P.E.} = -\vec{M} \cdot \vec{B}) \\ &= -\vec{M} \cdot \vec{B}_f + \vec{M} \cdot \vec{B}_i + 0 \\ &= -MB \cos 90^\circ + MB \cos 0^\circ \\ &= MB \\ &= NIAB \\ &= 200 \times 100 \times 10^{-6} \times \frac{5}{2} \times 10^{-4} \times 1 = 5 \mu\text{J} \end{aligned}$$

59. A particle is doing simple harmonic motion of amplitude  $0.06 \text{ m}$  and time period  $3.14 \text{ s}$ . The maximum velocity of the particle is \_\_\_\_\_  $\text{cm/s}$ .

Ans. (12)

Sol. We know

$$\begin{aligned} v_{\text{max}} &= \omega A \quad \text{at mean position} \\ &= \frac{2\pi}{T} A = \frac{2\pi}{\pi} \times 0.06 = 0.12 \text{ m/sec} \\ v_{\text{max}} &= 12 \text{ cm/sec} \end{aligned}$$

60. For three vectors  $\vec{A} = (-x\hat{i} - 6\hat{j} - 2\hat{k})$ ,  $\vec{B} = (-\hat{i} + 4\hat{j} + 3\hat{k})$  and  $\vec{C} = (-8\hat{i} - \hat{j} + 3\hat{k})$ , if  $\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$ , then value of  $x$  is \_\_\_\_\_.

Ans. (4)

$$\text{Sol. } \vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 15\hat{i} - 21\hat{j} + 33\hat{k}$$

$$\begin{aligned} \vec{A} \cdot (\vec{B} \times \vec{C}) &= (-x\hat{i} - 6\hat{j} - 2\hat{k}) \cdot (15\hat{i} - 21\hat{j} + 33\hat{k}) \\ 0 &= -15x + 126 - 66 \\ 15x &= 60 \\ x &= 4 \end{aligned}$$

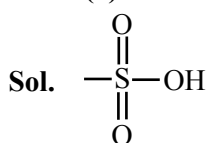
## CHEMISTRY

### SECTION-A

61. Functional group present in sulphonic acid is :

- (1)  $\text{SO}_4\text{H}$  (2)  $\text{SO}_3\text{H}$   
(3)  $\begin{array}{c} \text{O} \\ \parallel \\ -\text{S}-\text{OH} \end{array}$  (4)  $-\text{SO}_2$

Ans. (2)



Group present in sulphonic acids

62. Match List I with List II :

List I (Molecule / Species)		List II (Property / Shape)	
A.	$\text{SO}_2\text{Cl}_2$	I.	Paramagnetic
B.	$\text{NO}$	II.	Diamagnetic
C.	$\text{NO}_2^-$	III.	Tetrahedral
D.	$\text{I}_3^-$	IV.	Linear

Choose the **correct** answer from the options given below :

- (1) A-IV, B-I, C-III, D-II  
(2) A-III, B-I, C-II, D-IV  
(3) A-II, B-III, C-I, D-IV  
(4) A-III, B-IV, C-II, D-I

Ans. (2)

Sol.

(A)	$\text{SO}_2\text{Cl}_2$	$\text{sp}^3$	$\begin{array}{c} \text{O} \\ \parallel \\ \text{S} \\ \diagup \quad \diagdown \\ \text{O} \quad \text{Cl} \end{array}$ Tetrahedral
(B)	$\text{NO}$		Paramagnetic
(C)	$\text{NO}_2^-$		Diamagnetic
(D)	$\text{I}_3^-$	$\text{sp}^3\text{d}$	$\begin{array}{c} \text{I} \\   \\ \text{I} \\   \\ \text{I} \end{array}$ Linear

## TEST PAPER WITH SOLUTION

63. Given below are two statements :

**Statement I** : Picric acid is 2, 4, 6-trinitrotoluene.

**Statement II** : Phenol-2, 4-disulphuric acid is treated with conc.  $\text{HNO}_3$  to get picric acid.

In the light of the above statement, choose the **most appropriate** answer from the options given below :

- (1) Statement I is incorrect but Statement II is correct.  
(2) Both Statement I and Statement II are incorrect.  
(3) Statement I is correct but Statement II is incorrect.  
(4) Both Statement I and Statement II are correct.

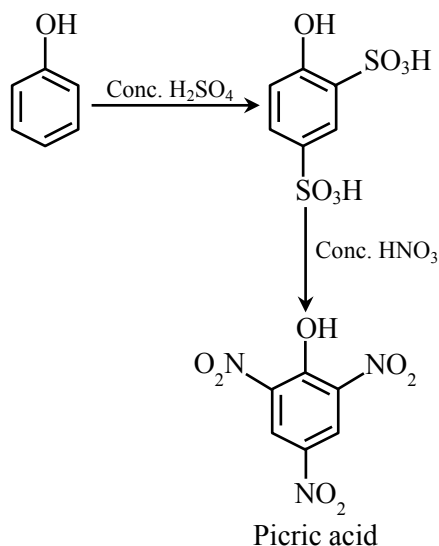
Ans. (1)

Sol.

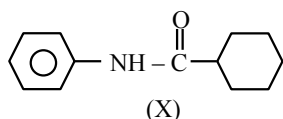


picric acid

(2, 4, 6 – trinitrophenol)



64. Which of the following is metamer of the given compound (X) ?

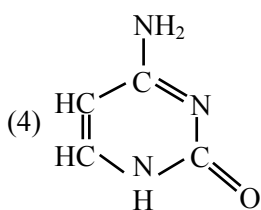
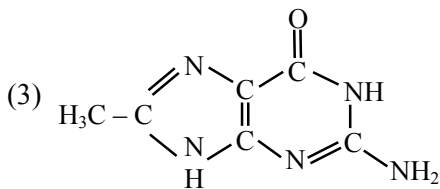
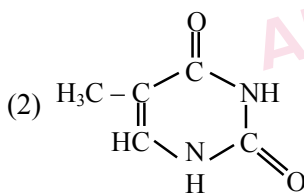
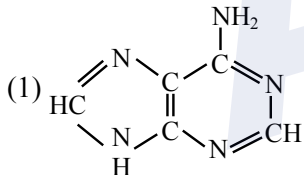


- (1)
- (2)
- (3)
- (4)

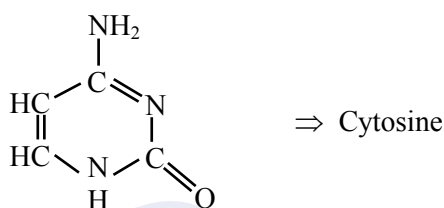
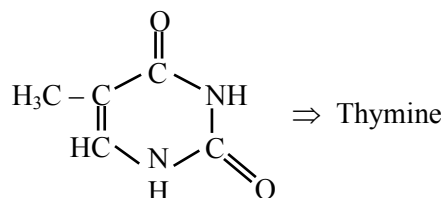
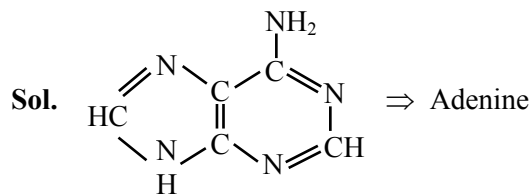
Ans. (4)

Sol. Metamer  $\Rightarrow$  Isomer having same molecular formula, same functional group but different alkyl/aryl groups on either side of functional group.

65. DNA molecule contains 4 bases whose structure are shown below. One of the structure is not correct, identify the **incorrect** base structure.



Ans. (3)



Are bases of DNA molecule. As DNA contain four bases, which are adenine, guanine, cytosine and thymine.

66. Match List I with List II :

LIST I (Hybridization)		LIST II (Orientation in Space)	
A.	$sp^3$	I.	Trigonal bipyramidal
B.	$dsp^2$	II.	Octahedral
C.	$sp^3d$	III.	Tetrahedral
D.	$sp^3d^2$	IV.	Square planar

Choose the **correct** answer from the options given below :

- (1) A-III, B-I, C-IV, D-II  
(2) A-II, B-I, C-IV, D-III  
(3) A-IV, B-III, C-I, D-II  
(4) A-III, B-IV, C-I, D-II

Ans. (4)

Sol.  $sp^3 \rightarrow$  Tetrahedral  
 $dsp^2 \rightarrow$  Square planar  
 $sp^3d \rightarrow$  Trigonal Bipyramidal  
 $sp^3d^2 \rightarrow$  Octahedral

67. Given below are two statements :

**Statement I** : Gallium is used in the manufacturing of thermometers.

**Statement II** : A thermometer containing gallium is useful for measuring the freezing point (256 K) of brine solution.

In the light of the above statement, choose the correct answer from the options given below :

- (1) Both Statement I and Statement II are false.
- (2) Statement I is false but Statement II is true.
- (3) Both Statement I and Statement II are true.
- (4) Statement I is true but Statement II is false.

**Ans. (4)**

**Sol. Statement - I**  $\Rightarrow$  Correct

**Statement - II**  $\Rightarrow$  False

Ga is used to measure high temperature

68. Which of the following statements are correct ?

- A. Glycerol is purified by vacuum distillation because it decomposes at its normal boiling point.
- B. Aniline can be purified by steam distillation as aniline is miscible in water.
- C. Ethanol can be separated from ethanol water mixture by azeotropic distillation because it forms azeotrope.
- D. An organic compound is pure, if mixed M.P. is remained same.

Choose the **most appropriate** answer from the options given below :

- (1) A, B, C only
- (2) A, C, D only
- (3) B, C, D only
- (4) A, B, D only

**Ans. (2)**

**Sol.** Option (B) is incorrect because aniline is immiscible in water.

69. Match List I with List II :

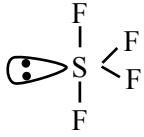
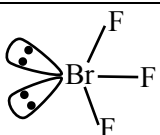
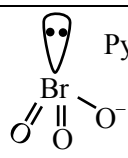
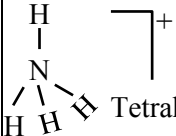
LIST I (Compound / Species)		LIST II (Shape / Geometry)	
A.	SF <sub>4</sub>	I.	Tetrahedral
B.	BrF <sub>3</sub>	II.	Pyramidal
C.	BrO <sub>3</sub> <sup>-</sup>	III.	See saw
D.	NH <sub>4</sub> <sup>+</sup>	IV.	Bent T-shape

Choose the **correct** answer from the options given below :

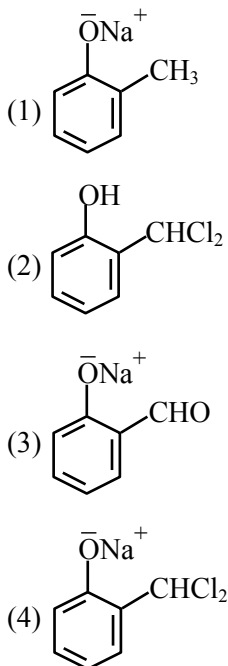
- (1) A-II, B-III, C-I, D-IV
- (2) A-III, B-IV, C-II, D-I
- (3) A-II, B-IV, C-III, D-I
- (4) A-III, B-II, C-IV, D-I

**Ans. (2)**

**Sol.**

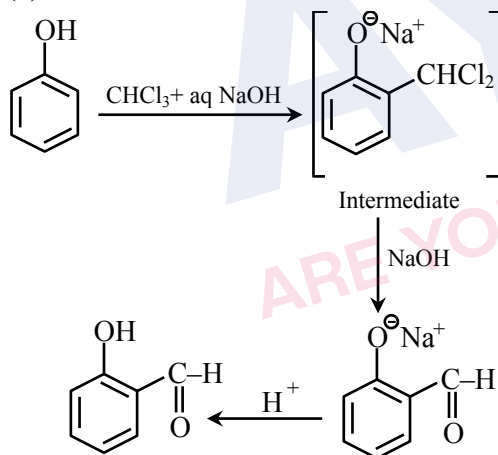
(A)	SF <sub>4</sub>	sp <sup>3</sup> d hybridisation	
(B)	BrF <sub>3</sub>	sp <sup>3</sup> d hybridisation	 Bent T-Shape
(C)	BrO <sub>3</sub> <sup>-</sup>	sp <sup>3</sup> hybridisation	 Pyramidal
(D)	NH <sub>4</sub> <sup>+</sup>	sp <sup>3</sup> hybridisation	 Tetrahedral

70. In Reimer - Tiemann reaction, phenol is converted into salicylaldehyde through an intermediate. The structure of intermediate is \_\_\_\_\_.



Ans. (4)

Sol.



71. Which of the following material is not a semiconductor.

- (1) Germanium  
(2) Graphite  
(3) Silicon  
(4) Copper oxide

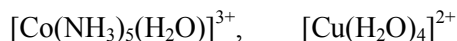
Ans. (2)

Sol. Graphite is conductor

72. Consider the following complexes.



(A) (B)



(C) (D)

The correct order of A, B, C and D in terms of wavenumber of light absorbed is :

- (1)  $C < D < A < B$   
(2)  $D < A < C < B$   
(3)  $A < C < B < D$   
(4)  $B < C < A < D$

Ans. (2)

Sol. As ligand field increases, light of more energy is absorbed

Energy  $\propto$  wave number

(v)

73. Match List I with List II :

LIST I (Precipitating reagent and conditions)		LIST II (Cation)	
A.	$\text{NH}_4\text{Cl} + \text{NH}_4\text{OH}$	I.	$\text{Mn}^{2+}$
B.	$\text{NH}_4\text{OH} + \text{Na}_2\text{CO}_3$	II.	$\text{Pb}^{2+}$
C.	$\text{NH}_4\text{OH} + \text{NH}_4\text{Cl} + \text{H}_2\text{S gas}$	III.	$\text{Al}^{3+}$
D.	dilute HCl	IV.	$\text{Sr}^{2+}$

Choose the correct answer from the options given below :

- (1) A-IV, B-III, C-II, D-I  
(2) A-IV, B-III, C-I, D-II  
(3) A-III, B-IV, C-I, D-II  
(4) A-III, B-IV, C-II, D-I

Ans. (3)

Sol. Theory based question

74. The electron affinity value are negative for :

- A.  $\text{Be} \rightarrow \text{Be}^-$   
B.  $\text{N} \rightarrow \text{N}^-$   
C.  $\text{O} \rightarrow \text{O}^{2-}$   
D.  $\text{Na} \rightarrow \text{Na}^-$   
E.  $\text{Al} \rightarrow \text{Al}^-$

Choose the most appropriate answer from the options given below :

- (1) D and E only (2) A, B, D and E only  
(3) A and D only (4) A, B and C only

**Allen Ans. (4)**

**NTA Ans. (1)**

- Sol.** (A)  $\text{Be} + e^- \rightarrow \text{Be}^-$ , E.A = -ive  
(B)  $\text{N} + e^- \rightarrow \text{N}^-$  E.A = -ive  
(C)  $\text{O} + e^- \rightarrow \text{O}^-$   
 $\text{O}^- + e^- \rightarrow \text{O}^{2-}$  E.A = -ive  
(D)  $\text{Na} + e^- \rightarrow \text{Na}^-$  E.A = +ive  
(E)  $\text{Al} + e^- \rightarrow \text{Al}^-$  E.A = +ive

Ans. A, B and C only

75. The number of element from the following that do not belong to lanthanoids is :

Eu, Cm, Er, Tb, Yb and Lu

- (1) 3 (2) 4  
(3) 1 (4) 5

**Ans. (3)**

**Sol.** Cm is Actinide

76. The density of 'x' M solution ('x' molar) of NaOH is  $1.12 \text{ g mL}^{-1}$ . while in molality, the concentration of the solution is 3 m (3 molal). Then x is (Given : Molar mass of NaOH is 40 g/mol)

- (1) 3.5 (2) 3.0  
(3) 3.8 (4) 2.8

**Ans. (2)**

**Sol.** Molality =  $\frac{1000 \times M}{1000 \times d - M \times (Mw)_{\text{solute}}}$

$$3 = \frac{1000 \times x}{1000 \times 1.12 - (x \times 40)}$$

$$x = 3$$

77. Which among the following aldehydes is most reactive towards nucleophilic addition reactions?

- (1)  $\text{H}-\overset{\text{O}}{\parallel}{\text{C}}-\text{H}$  (2)  $\text{C}_2\text{H}_5-\overset{\text{O}}{\parallel}{\text{C}}-\text{H}$   
(3)  $\text{CH}_3-\overset{\text{O}}{\parallel}{\text{C}}-\text{H}$  (4)  $\text{C}_3\text{H}_7-\overset{\text{O}}{\parallel}{\text{C}}-\text{H}$

**Ans. (1)**

**Sol.**  $\text{H}-\overset{\text{O}}{\parallel}{\text{C}}-\text{H}$  has low steric hindrance at carbonyl carbon and high partial positive charge at carbonyl carbon.

78. At  $-20^\circ\text{C}$  and 1 atm pressure, a cylinder is filled with equal number of  $\text{H}_2$ ,  $\text{I}_2$  and HI molecules for the reaction

$\text{H}_2(\text{g}) + \text{I}_2(\text{g}) \rightleftharpoons 2\text{HI}(\text{g})$ , the  $K_p$  for the process is  $x \times 10^{-1}$ .  $x =$  \_\_\_\_\_.

[Given :  $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$ ]

- (1) 2 (2) 1  
(3) 10 (4) 0.01

**Ans. (3)**

**Sol.**  $\Delta n_g = 0$   $K_p = \frac{(n_{\text{HI}})^2}{n_{\text{H}_2} n_{\text{I}_2}} \left( \frac{P_T}{n_T} \right)^{\Delta n_g}$   
 $n_{\text{HI}} = n_{\text{H}_2} = n_{\text{I}_2}$  so  $K_p = 1$   
 $1 = x \times 10^{-1}$   $x = 10$

79. Match List I with List II :

LIST I (Compound)		LIST II (Uses)	
A.	Iodoform	I.	Fire extinguisher
B.	Carbon tetrachloride	II.	Insecticide
C.	CFC	III.	Antiseptic
D.	DDT	IV.	Refrigerants

Choose the **correct** answer from the options given below :

- (1) A-I, B-II, C-III, D-IV  
(2) A-III, B-II, C-IV, D-I  
(3) A-III, B-I, C-IV, D-II  
(4) A-II, B-IV, C-I, D-III

**Ans. (3)**



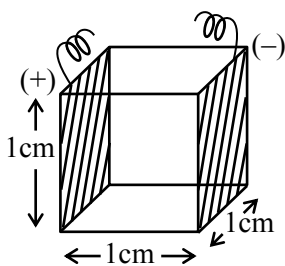
**Sol.** Iodoform – Antiseptic

$\text{CCl}_4$  – Fire extinguisher

CFC – Refrigerants

DDT – Insecticide

- 80.** A conductivity cell with two electrodes (dark side) are half filled with infinitely dilute aqueous solution of a weak electrolyte. If volume is doubled by adding more water at constant temperature, the molar conductivity of the cell will -



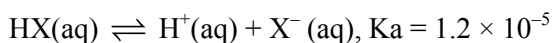
- (1) increase sharply
- (2) remain same or can not be measured accurately
- (3) decrease sharply
- (4) depend upon type of electrolyte

**Ans. (2)**

**Sol.** Solution is already infinitely dilute, hence no change in molar conductivity upon addition of water

### SECTION-B

- 81.** Consider the dissociation of the weak acid HX as given below



[ $K_a$  : dissociation constant]

The osmotic pressure of 0.03 M aqueous solution of HX at 300 K is  $\text{---} \times 10^{-2}$  bar (nearest integer).

[Given :  $R = 0.083 \text{ L bar Mol}^{-1} \text{ K}^{-1}$ ]

**Ans. (76)**

**Sol.**  $\text{HX} \rightleftharpoons \text{H}^+ + \text{X}^- \quad K_a = 1.2 \times 10^{-5}$

0.03M

0.03 - x      x      x

$$K_a = 1.2 \times 10^{-5} = \frac{x^2}{0.03 - x}$$

0.03 - x  $\approx$  0.03 ( $K_a$  is very small)

$$\frac{x^2}{0.03} = 1.2 \times 10^{-5}$$

$$x = 6 \times 10^{-4}$$

Final solution : 0.03 - x + x + x

$$= 0.03 + x = 0.03 + 6 \times 10^{-4}$$

$$\Pi = (0.03 + (6 \times 10^{-4})) \times 0.083 \times 300$$

$$= 76.19 \times 10^{-2} \approx 76 \times 10^{-2}$$

- 82.** The difference in the 'spin-only' magnetic moment values of  $\text{KMnO}_4$  and the manganese product formed during titration of  $\text{KMnO}_4$  against oxalic acid in acidic medium is  $\text{---}$  BM. (nearest integer)

**Ans. (6)**

**Sol.** Spin only magnetic moment of Mn in  $\text{KMnO}_4 = 0$   
Spin only value of manganese product formed during titration of  $\text{KMnO}_4$  against oxalic acid in acidic medium is = 6

Ans. 6

- 83.** Time required for 99.9% completion of a first order reaction is  $\text{---}$  time the time required for completion of 90% reaction. (nearest integer).

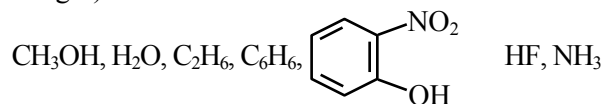
**Ans. (3)**

$$\text{Sol. } K = \frac{1}{t_{99.9\%}} \ln \left( \frac{100}{0.1} \right) = \frac{1}{t_{90\%}} \ln \left( \frac{100}{10} \right)$$

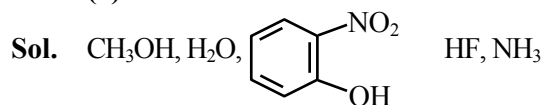
$$t_{99.9\%} = t_{90\%} \frac{\ln(10^3)}{\ln 10}$$

$$t_{99.9\%} = t_{90\%} \times 3$$

- 84.** Number of molecules from the following which can exhibit hydrogen bonding is  $\text{---}$ . (nearest integer)



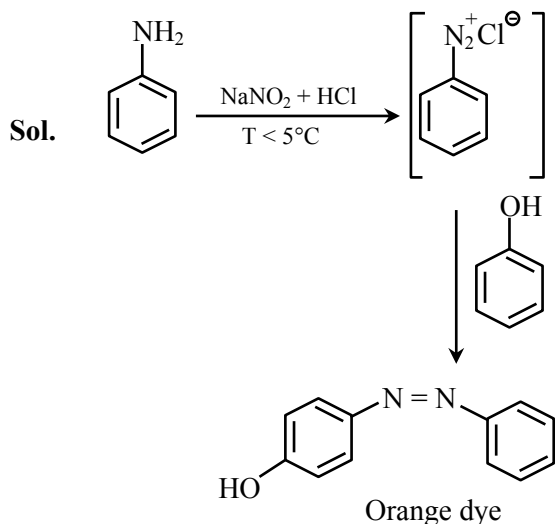
**Ans. (5)**



Can show H-bonding.

85. 9.3 g of pure aniline upon diazotisation followed by coupling with phenol gives an orange dye. The mass of orange dye produced (assume 100% yield/ conversion) is \_\_\_\_\_ g. (nearest integer)

Ans. (20)



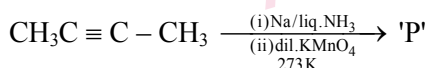
Reaction suggests that 1 mole of aniline give 1 mole of orange dye.

so (mol)<sub>aniline</sub> = (mole)<sub>orange dye</sub>

$$\frac{9.3\text{g}}{93\text{g mol}^{-1}} = \frac{\text{mass of orange dye}}{199\text{g mol}^{-1}}$$

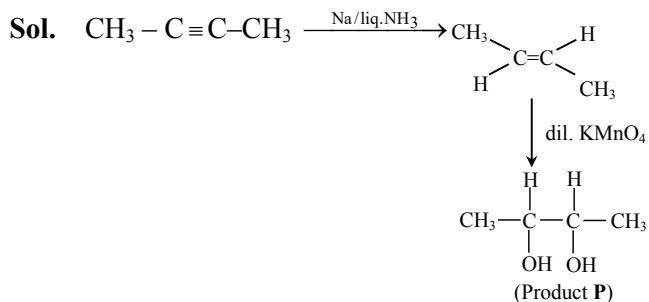
mass of orange dye = 19.9 g  $\approx$  20 g

86. The major product of the following reaction is P.



Number of oxygen atoms present in product 'P' is \_\_\_\_\_ (nearest integer).

Ans. (2)



87. Frequency of the de-Broglie wave of electron in Bohr's first orbit of hydrogen atom is \_\_\_\_\_  $\times 10^{13}$  Hz (nearest integer).

[Given :  $R_H$  (Rydberg constant) =  $2.18 \times 10^{-18}$  J.  
 $h$  (Plank's constant) =  $6.6 \times 10^{-34}$  J.s.]

Allen Ans. (661)

NTA Ans. (658)

Sol.  $\lambda = \frac{h}{mv}$

$$\lambda = \frac{h\nu}{mv^2}$$

$$\frac{mv^2}{h} = \frac{\nu}{\lambda} = \nu \text{ (frequency)}$$

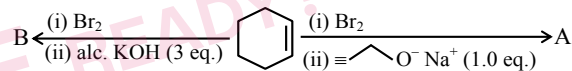
$$\text{Given } \frac{1}{2} mv^2 = 2.18 \times 10^{-18} \text{ J}$$

$$h = 6.6 \times 10^{-34}$$

$$\nu = \frac{4.36 \times 10^{-18}}{6.6 \times 10^{-34}} = 660.60 \times 10^{13} \text{ Hz}$$

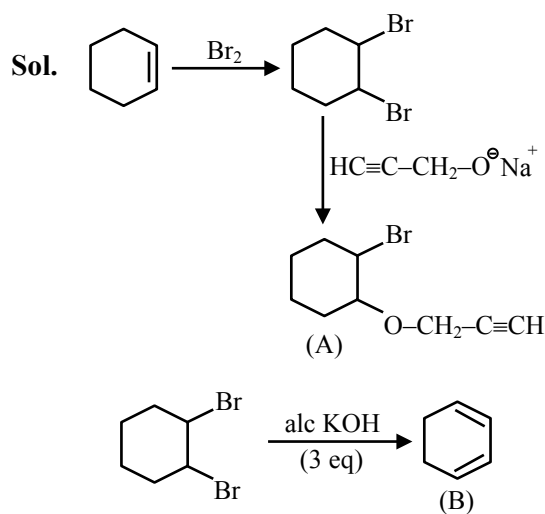
$$\approx 661 \times 10^{13} \text{ Hz}$$

88. The major products from the following reaction sequence are product A and product B.



The total sum of  $\pi$  electrons in product A and product B are \_\_\_\_\_ (nearest integer)

Ans. (8)



89. Among CrO, Cr<sub>2</sub>O<sub>3</sub> and CrO<sub>3</sub>, the sum of spin-only magnetic moment values of basic and amphoteric oxides is \_\_\_\_\_  $10^{-2}$  BM (nearest integer).

(Given atomic number of Cr is 24)

**Ans. (877)**

**Sol.** CrO Basic oxide

Cr<sub>2</sub>O<sub>3</sub> Amphoteric oxide

In CrO, Cr exist as Cr<sup>+2</sup> and have  $\mu$  only = 4.90

In Cr<sub>2</sub>O<sub>3</sub>, Cr exist as Cr<sup>+3</sup> and have  $\mu$  only = 3.87

Sum of spin only magnetic moment

$$= 4.90 + 3.87 = 8.77$$

$$\mu_{\text{only}} = 877 \times 10^{-2}$$

Ans. 877

90. An ideal gas,  $\bar{C}_v = \frac{5}{2}R$ , is expanded adiabatically against a constant pressure of 1 atm until it doubles in volume. If the initial temperature and pressure is 298 K and 5 atm, respectively then the final temperature is \_\_\_\_\_ K (nearest integer).

[ $\bar{C}_v$  is the molar heat capacity at constant volume]

**Ans. (274)**

**Sol.**  $\Delta U = q + w$  ( $q = 0$ )

$$nC_v \Delta T = -P_{\text{ext}} (V_2 - V_1)$$

$$V_2 = 2V_1$$

$$\frac{nRT_2}{P_2} = \frac{2nRT_1}{P_1}$$

$$P_1 = 5, T_1 = 298$$

$$P_2 = \frac{5T_2}{2 \times 298}$$

$$n \frac{5}{2} R(T_2 - T_1) = -1 \left( \frac{nRT_2}{P_1} - \frac{nRT_1}{P_1} \right)$$

$$\text{Put } T_1 = 298$$

$$\text{and } P_2 = \frac{5T_2}{2 \times 298}$$

$$\text{Solve and we get } T_2 = 274.16 \text{ K}$$

$$T_2 \approx 274 \text{ K}$$